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Non-Convex and Convex Coupling Image Segmentation via TGpV Regularization and Thresholding

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Abstract. In this paper, we propose a non-convex and convex coupling variational model for image segmentation. We design the non-convex and convex regularization terms based on total generalized p-variation (TGpV) regularizer to preserve the boundary of segmented parts and detect the structure in the image. Our method has two stages. The first stage is to approximate the Mumford-Shah model. The second stage is to segment the smoothed u into different phases by using a thresholding strategy. We develop a scheme based on the alternating direction method of multipliers (ADMM) algorithm, generalized p-shrinkage operation and K-means clustering method to carry out our method. We perform numerical experiments on many kinds of images such as real Bacteria image, Tubular magnetic resonance angiography (MRA) image, magnetic resonance (MR) images, anti-mass images, artificial images, noisy or blurred images. Some comparisons are arranged to show the effectiveness and advantages of our method.

AMS subject classifications: 65K10, 68U10, 90C26

Key words: Two-stage strategy, non-convex and convex coupling, total generalized p-variation (TGpV), alternating direction method of multipliers (ADMM), clustering methods.

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1 Introduction

Image segmentation is of great importance in image processing and computer vision with various applications, such as object detection and medical imaging [23, 65]. The goal of segmentation is to divide the image into regions that belong to distinct objects in the depicted scene. Over these years, approaches of image processing based on the calculus of variations and partial differential equations (PDEs) have been extensively

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studied [9, 25, 41, 52]. Especially, many successful methods for image segmentation are based on variational models. Generally, these models can be divided into two categories: edge-based image segmentation models and region-based image segmentation models. Mumford-Shah (MS) model is one of the most popular and influential region-based image segmentation models, which is a non-convex variational model and pursues a piecewise constant or smooth approximation of the given image [41].

Since the MS model itself is non-convex and non-smooth, it is tough to find its minimizer. Consequently, many models and algorithms have been widely studied to solve the tough optimization problem [1,10,11,27,40]. For example, in [1] Ambrosio et al. used a phase field energy to replace the length of Γ and used a sequence of simpler elliptic variational problems to approximate the MS model. And then, some non-local approximation methods [11, 40] were proposed by using a family of continuous functions to avoid computing Γ explicitly. Meanwhile, many people have tried to simplify the MS model. One of the famous examples is the Chan-Vese (CV) model [16], they restricted the solution to be two constants, we call this type of model as piecewise constant MS model. More works and details for the general piecewise constant MS model can be found in [35, 52–54, 56], etc. Literally, the MS model and CV model have been extended to process color images [15], multiphase images [56], and piecewise-smooth images [34]. More recently, another class of piecewise-smooth segmentation models was introduced in [9, 24, 36, 49], based on the level-set representation and using local feature information. Despite the advantages of the level-set based methods [50, 51], solutions can easily become trapped in local minima depending on the initialization, and moreover, they are rather slow compared to related methods. Many models such as graph cut models [5,30] and convex relaxation models [7,9,14] were proposed to overcome the numerical difficulties of the non-convex problem. Especially, recently Cai et al. have proposed a two-stage scheme for image segmentation [9]. Based on the MS model, the framework has two stages: smoothing and thresholding. Furthermore, from the results in [9], we find that the superior segmentation results can be produced with fast and reliable numerical implementations.

In this paper, we propose the following non-convex and convex coupling variational segmentation model:

$$\min_u \frac{\eta}{2}\int_\Omega (f-\mathcal{A}u)^2 dx + \frac{\mu}{2}\int_\Omega |\nabla u|^2 dx + \mathrm{TGpV}_\alpha^2(u), \tag{1.1}$$

where η and μ are positive weighted parameters controlling the contribution of each term, $\mathrm{TGpV}_\alpha^2(u)$ is a non-convex regularizer; detailed definition in Section 2. In our model, $\mathcal{A}$ could stand for the identity operator or a blur operator (if the image is corrupted by blur).

The idea of designing the regularization terms containing non-convex (sparsity-promoting) terms, referred to as CNC strategy was first introduced by Blake and Zisserman in [3], and was studied widely for different purposes, see [13, 22, 43, 47]. The idea of coupling has been widely used in many papers [32,42,63]. Especially, in [13] Chan et al.