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Fitted Finite Volume Method for Pricing American Options under Regime-Switching Jump-Diffusion Models Based on Penalty Method

Xiaoting Gan[1,2,*], Junfeng Yin[3] and Rui Li[4]

[1] *School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu 611731, Sichuan, China*
[2] *School of Mathematics and Statistics, Chuxiong Normal University, Chuxiong 675000, Yunnan, China*
[3] *School of Mathematical Sciences, Tongji University, Shanghai 200092, China*
[4] *College of Mathematics Physics and Information Engineering, Jiaxing University, Jiaxing 314001, Zhejiang, China*

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Abstract. In this paper we develop a novel numerical method for pricing American options under regime-switching jump-diffusion models which are governed by a system of partial integro-differential complementarity problems (PIDCPs). Based on a penalty approach, the PIDCPs results in a set of coupled nonlinear partial integro-differential equations (PIDEs). To numerically solve these nonlinear penalized PIDEs, we introduce a fitted finite volume method for the spatial discretization, coupled with the backward Euler and Crank-Nicolson time stepping schemes. We show that these schemes are consistent, stable and monotone, hence it ensures the convergence to the solution of continuous problem. To solve the discretized nonlinear system effectively, an iterative method is designed. Numerical experiments are presented to demonstrate the accuracy, efficiency and robustness of the new numerical method.

AMS subject classifications: 65M06, 65M12, 65M32, 91G60

Key words: American option pricing, regime-switching jump-diffusion model, complementarity problem, fitted finite volume method, Penalty method.

1 Introduction

It is well-known that the standard Black-Scholes model has been successfully and widely used in finance [1]. However, empirical findings that the classical Black-Scholes model cannot account for the phenomenon of volatility smiles occurs in all major stock markets.

*Corresponding author.
Emails: 9xtgan@alumni.tongji.edu.cn (X. T. Gan), yinjf@tongji.edu.cn (J. F. Yin), lirui@tongji.edu.cn (R. Li)

To remedy this defect, researchers have made various extensions to this model. Among of them, the regime-switching model [2, 3], in which the model parameters (drift and volatility coefficients) are assumed to depend on a Markov chain, is getting more and more attractive both in financial study and industry [4]. Actually, the regime-switching framework allows one to account for certain periodic or cyclic patterns caused by, e.g., short-term political or economic uncertainty. Hence, the regime-switching model can better fit the market dynamics.

In this paper, we focus on pricing American options under regime-switching jump-diffusion models. More specifically, we concentrate on the numerical solution of the American options model which can be formulated as a parabolic PIDCP with suitable boundary and terminal conditions [5]. Unlike the European options pricing, due to the early exercise feature of the American options, the closed-form solution generally does not exit. Hence, numerical approximation methods are normally employed to solve them [6]. Especially, the American options under regime-switching jump-diffusion models are much harder to be solved numerically because they require to handle a system of coupled PIDCPs simultaneously. In the past few years, for the valuation of American options under regime-switching jump-diffusion models, there are several numerical methods developed to solve a system of PIDCPs. In [5], the implicit finite difference method with three time levels coupled with the operator splitting method was presented to numerically solve the American options pricing problem. Two iterative algorithms were designed in [7] for the European options pricing problem. Bastani et al. [8] employed a radial basis collocation method based on a mesh-free approach for the American option pricing problem. An explicit formula and a more general multinomial approach proposed by Costabile et al. in [9] for pricing European and American options. In [10], a high order finite element scheme using the Lagrange techniques and the exponential time integration methods were developed to evaluate European, American and Butterfly options. A fourth-order compact finite difference scheme with three time levels along with operator splitting technique was proposed to solve the American option pricing problem by Patel et al. in [11]. A front-fixing finite element method was studied in [12] for pricing American options. Kazmi et al. developed a predictor-corrector nature of the L-stable method and presented an IMEX predictor-corrector method for pricing both European and American options in [13] and [14], respectively. Based on the penalty approach, in [15], Zhang et al. developed a fitted finite volume method for pricing American options under regime-switching model without the inclusion of jumps. Meanwhile, by virtue of variational inequality theory, a power penalty approach further studied by Zhang et al. for this American options pricing problem in [16].

The fitted finite volume method was first used to price standard European options by Wang in [17], and then generalized to other types of options by various authors, see e.g., [18–26] and the reference therein. Generally speaking, this method is based on a popular exponentially fitting technique and widely used for the boundary and interior layers problems [27, 28]. It has been shown that fitted finite volume method makes greater success in options pricing because it can overcome the difficulty caused by drift-dominated