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A New Second-Order One-Step Scheme for Solving Decoupled FBSDES and Optimal Error Estimates

Yang Li[1], Jie Yang[2] and Weidong Zhao[3,*]

[1]*College of Science, University of Shanghai for Science and Technology, Shanghai 200093, China.*
[2]*School of Mathematics and Statistics, Shandong University, Weihai, Shandong 264209, China.*
[3]*School of Mathematics, Shandong University, Jinan, Shandong 250100, China.*

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Abstract. A novel second-order numerical scheme for solving decoupled forward backward stochastic differential equations is proposed. Unlike known second-order schemes for such equations, the forward stochastic differential equations are approximated by a simplified weak order-2 Itô-Taylor scheme. This makes the method more implementable and enhances the accuracy. If the operators involved satisfy certain commutativity conditions, the scheme with quadratic convergence can be simplified, which is important in applications. The stability of the method is studied and second-order optimal error estimates are obtained.

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Key words: FBSDEs, simplified weak Itô-Taylor scheme, second-order scheme, error estimate.

1. Introduction

Let $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{0\le t\le T},P)$ be a complete filtered probability space with the filtration $\{\mathscr{F}_t\}_{0\le t\le T}$ generated by a standard d-dimensional Brownian motion $W_t=(W_t^1,\dots,W_t^d)^\top$, $t\ge 0$. This work is devoted to numerical solution of forward backward stochastic differential equations (FBSDEs) on the probability space $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{0\le t\le T},P)$, viz.

$$\begin{aligned} X_t &= X_0+\int_0^t b(s,X_s)ds+\int_0^t \sigma(s,X_s)dW_s \qquad \text{(SDE)},\\ Y_t &= \varphi(X_T)+\int_t^T f(s,X_s,Y_s,Z_s)ds-\int_t^T Z_s dW_s \quad \text{(BSDE)}, \end{aligned} \tag{1.1}$$

*Corresponding author. *Email addresses:* `wdzhao@sdu.edu.cn` (W. Zhao).

where $t \in [0, T]$, $T > 0$ is a deterministic terminal time, $X_0 \in \mathscr{F}_0$ the initial condition of the forward stochastic differential equation (SDE), $f : [0, T] \times \mathbb{R}^d \times \mathbb{R} \times \mathbb{R}^{1\times d} \to \mathbb{R}$ the generator of the backward stochastic differential equation (BSDE), and $\varphi : \mathbb{R}^d \to \mathbb{R}$ the terminal condition. The terms $b : [0, T] \times \mathbb{R}^d \to \mathbb{R}^d$ and $\sigma : [0, T] \times \mathbb{R}^d \to \mathbb{R}^{d\times d}$ are, respectively, called drift and diffusion coefficients of SDE.

A triplet process $(X_t, Y_t, Z_t) : \Omega \times [0, T] \to \mathbb{R}^d \times \mathbb{R} \times \mathbb{R}^{1\times d}$ is called an L^2-adapted solution of (1.1) if it is $\mathscr{F}_t$-adapted, square integrable and satisfies the FBSDEs (1.1).

Pardoux and Peng [15] established the unique solvability of general nonlinear backward stochastic differential equations (BSDEs). Such equations found numerous applications in stochastic control, mathematical finance, risk measurement, partial differential equations (PDEs), nonlinear expectations — cf. Refs. [7, 9, 12, 17, 18] and are vigorously studied [11,12,23,25,27]. However, since analytic solutions of FBSDEs are rarely known, they are often approximated by numerical methods [4–6, 8, 20–30, 32, 33]. In particular, second-order one-step numerical schemes for decoupled FBSDEs are studied in [3, 28, 30, 33]. Thus Crisan and Manolarakis [3] considered a second-order scheme by introducing new Gaussian random variables but they did not discretise forward SDE. To enhance the stability of numerical schemes for FBSDEs, one-step second-order numerical methods using new Gaussian random variables are proposed in [4, 33]. The accuracy and the effectiveness of these methods depend on Gaussian random variables and on the methods used in solving forward SDEs.

There is a vast literature on numerical methods for forward SDEs, including strong and weak Itô-Taylor schemes, Euler scheme, Milstein scheme [10, 11, 29, 30, 33]. Most of the approaches incorporate multiple stochastic integrals with respect to Brownian motions. However, the use of stochastic integrals leads to implementation problems, especially in multi-dimensional situation. On the other hand, Kloeden and Platen [10] developed an efficient method for the approximation of multiple stochastic integrals, called simplified weak order Itô-Taylor schemes for SDEs. Relatively simple second-order schemes for FBSDEs, which use a Gaussian process $\Delta\tilde{W}_s$, are proposed and studied in [3, 4, 33]. They have the second order convergence. Nevertheless, the presence of a Gaussian process diminishes the convergence rates of simplified weak order-2 Itô-Taylor scheme for forward SDE. In fact, these schemes would have the first-order accuracy.

In this work, we use a nonlinear Feynman-Kac formula and Malliavin calculus to investigate the weak Itô-Taylor schemes of order two and to develop a new one-step second-order scheme for the FBSDEs (1.1). In particular, a simplified weak order two Taylor scheme is used to solve the forward SDE in the FBSDEs. In comparison with known second-order numerical schemes in [2, 3, 11, 30, 33], this approach involves no multiple stochastic integrals, which makes it more efficient and more implementable for multidimensional FBSDEs. Moreover, under certain regularity conditions on the coefficients b, σ, the generator f and the terminal condition φ, the stability and the second-order convergence of the scheme with respect to time partition Δt are rigorously proved. The main features of the new scheme are:

- It involves no multiple stochastic Itô-type integrals and can be easily applied to multidimensional FBSDEs.