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Convergence of Linear Multistep Methods and One-Leg Methods for Index-2 Differential-Algebraic Equations with a Variable Delay

Hongliang Liu[1] and Aiguo Xiao[1,*]

[1] *School of Mathematics and Computational Science, Key Laboratory of Intelligent Computing and Information Processing of Ministry of Education, Xiangtan University, Xiangtan, Hunan 411105, P. R. China*

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Abstract. Linear multistep methods and one-leg methods are applied to a class of index-2 nonlinear differential-algebraic equations with a variable delay. The corresponding convergence results are obtained and successfully confirmed by some numerical examples. The results obtained in this work extend the corresponding ones in literature.

AMS subject classifications: 65L80; 65L20

Key words: index-2 differential-algebraic equations, variable delay, linear mutistep methods, one-leg methods, convergence.

1 Introduction

Index-2 delay differential-algebraic equations (DDAEs) are a very important class of mathematical models and often arise from the fields of computer aided design, circuit analysis, mechanical system, etc. Hence, the study of numerical methods for these equations is of important theoretical and practical values. In the recent years, some researches have been devoted to numerical methods for differential algebraic equations [1–7]. Some stability and convergence results of numerical methods for linear or index-1 delay differential-algebraic equations have been presented [8–11]. Xu and Zhao [8] studied stability of Runge-Kutta methods for neutral delay integro differential-algebraic equations. Block implicit one-step methods were applied to a class of retarded differential-algebraic equations by Li [9]. Convergence of one-leg methods for index-1 delay differential-algebraic equations was proved by Xiao and Zhang [10]. Zhu and Petzold [11] discussed asymptotic stability of Hessenberg delay differential-algebraic equations. However, the researches into numerical methods

*Corresponding author.
Email: lhl@xtu.edu.cn (H. Liu), xag@xtu.edu.cn (A. Xiao)

for nonlinear high-index delay differential-algebraic equations have arised in a few references [12–14]. Ascher and Petzold [12] derived the classical convergence results of BDF methods and Runge-Kutta methods for index-2 constant-delay differential-algebraic equations. Hauber [13] applied collocation methods to retarded differential-algebraic equations. Liu and Xiao [14] obtained the convergence results of BDF methods for a class of index-2 differential-algebraic equations with a variable delay.

In this paper, we apply the linear multistep methods (LMMs) and one-leg methods to a class of index-2 nonlinear differential-algebraic equations with a variable delay. The corresponding convergence results are obtained and successfully confirmed by some numerical examples.

2 Convergence of linear multistep methods

Consider the semi-explicit index-2 DDAE

$$\begin{cases} y'(x) = f\Big(y(x), y(x-\tau(x)), z(x)\Big), & x \in [0,T], \\ 0 = g(y(x)), & x \in [0,T], \\ z(0) = z_0, \quad y(x) = \varphi(x), & x \in [-\tau, 0], \end{cases} \tag{2.1}$$

where delay function $\tau(x)$ is differentiable and satisfies $0 < \tau(x) \le \tau, \tau'(x) < 1$, $f : R^{n_1} \times R^{n_1} \times R^{n_2} \to R^{n_1}$, $g : R^{n_1} \to R^{n_2}$ are sufficiently smooth vector functions on the real Euclidean spaces and have bounded derivatives, the initial value function $\varphi : [-\tau, 0] \to R^{n_1}$ is a continuous function, and $g_y(y)f_z(y, y(x-\tau(x)), z)$ is invertible and bounded in a neighbourhood of the solution. We assume that the problem (2.1) has a smooth solution $y(x), z(x)$. Throughout this paper, $\|\cdot\|$ denotes the standard Euclidean norm, and the matrix norm is subordinate to $\|\cdot\|$.

A LMM with a Lagrange interpolation polynomial of degree p applied to the system (2.1) reads

$$\sum_{i=0}^{k} \alpha_i y_{n+i} = h \sum_{i=0}^{k} \beta_i f(y_{n+i}, y^h_{n-k+i}, z_{n+i}), \tag{2.2a}$$

$$0 = g(y_{n+k}), \tag{2.2b}$$

where $x_{n+i} = x_n + ih,\ n \ge 0$,

$$y^h_{n-k+i} = \begin{cases} \varphi(x_{n+i} - \tau(x_{n+i})), & x_{n+i} - \tau(x_{n+i}) \le 0, i = 0, 1, \cdots, k, \\ \sum\limits_{j=-u}^{q} Q_j(\delta_{n_i}) y_{n+i-m_{n_i}+j}, & x_{n+i} - \tau(x_{n+i}) > 0, i = 0, 1, \cdots, k, \end{cases} \tag{2.3}$$

where $\tau(x_{n+i}) = (m_{n_i} - \delta_{n_i})h,\ u, q, m_{n_i} \in Z^+,\ \delta_{n_i} \in [0, 1),\ q + u = p, q + 1 \le m_{n_i},\ Q_j(\delta_{n_i})$ is the Lagrange interpolation basic function.