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Runge-Kutta Finite Element Method Based on the Characteristic for the Incompressible Navier-Stokes Equations

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Abstract. In this paper, a finite element method based on the characteristic for the incompressible Navier-Stokes equations is proposed by introducing Runge-Kutta method. At first, coordinate transformation operation is performed to obtain the alternative Navier-Stokes equations without convection term. Then, instead of the classical characteristic-based split (CBS) method, we use the third-order Runge-Kutta method along the characteristic to carry out time discretization in order to improve calculation accuracy, and segregate the calculation of the pressure from that of the velocity based on the momentum-pressure Poisson equation method. Finally, some classical benchmark problems are used to validate the effectiveness of the present method. Compared with the classical method, the present method has lower dissipation, larger permissible time step, and higher time accuracy. The code can be downloaded at DOI: 10.13140/RG.2.2.36336.56329.

AMS subject classifications: 65M12, 76D05

Key words: Finite element method, characteristic, Navier-Stokes equations, Runge-Kutta method, accuracy.

1 Introduction

In the field of fluid dynamics, The incompressible Navier–Stokes (N-S) equations are used to model a number of important physical phenomena. However, as is well known,

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in FEM, as well as in FDM and FVM, the numerical solution of incompressible N-S equations with the standard Galerkin method leads to spurious oscillatory at high Reynolds number. So significant emphasis has been placed in the literature on developing stabilized and high-accuracy algorithm enough to simulate complex flows [1,2].

The numerical instability is due to the following three aspects. Firstly, the form of the convection term is non-self-adjoint, but the standard Galerkin method adopt a central-difference type approximation to the convection term. Secondly, in convection dominated flows, for which layers appears where the velocity solution and its gradient exhibit rapid variation, the standard Galerkin method leads to numerical oscillations in these layer regions which can spread quickly and pollute the entire solution domain [3]. Lastly, the use of inappropriate combinations of interpolation functions for the velocity and pressure yields unstable schemes [4].

Based on this, the stream upwind Petrov-Galerkin method (SUPG) [5], the Galerkin least square method (GLS) [6,7], the finite increment method (FIC) [8], the penalty function method [9,10] and the Taylor-Galerkin method [11,12] were proposed, which were recognized as the effective stabilization method of the pressure field. The idea of all these methods is based on adding a viscous dissipation term to the original Galerkin formulation of the governing equations, in order to suppress spurious oscillations efficiently. In these methods, the SUPG method [13,14] uses the asymmetric weight function to increase the weight of the inflow direction and reduce the weight of the outflow direction, and then obtain artificial viscous dissipation term. However, the optimal upwind coefficient is usually difficult to determine, and the element matrix needs to be updated at each iteration step, which reduces the computational efficiency. The characteristic Galerkin (CG) [11, 15] method discretes the particle time derivatives along the characteristic instead of the spatial time derivative, so the convection term disappears and the problem is that of simple diffusion, for which the standard Galerkin approximation is optimal.

In order to avoid the complex programming and time consuming of the CG method, Zienkiewicz and Codina [16,17] proposed the characteristic-based time splitting Galerkin method (CBS) and made foundation research. The CBS method takes advantage of local Taylor expansion along the characteristic to eliminate the convection term, and introduces an auxiliary variable velocity to decouple the pressure. Finally, based on the standard Galerkin method, the velocity and pressure field are obtained. The CBS method is well documented and easy to implement, More importantly, it can also use the same interpolation for both velocity and pressure to circumvent the LBB condition [18], so it has been applied widely for the solution of fluid and solid dynamic problems in recent years [19–21], including shallow water flow [22], flow past bluff-bodies [23,24] and porous medium flow [25].

It is worth noting that the CBS method has only first order accuracy and needs to introduce an auxiliary variable velocity, so many researchers develop and improve it from different aspects. By retaining the pressure gradient in the split step and artificial compressibility (AC) parameter, Nithiarasu [26] and He [27] improved the CBS method to second-order accuracy, but the AC parameter needs to be determined empirically.