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Approximate Solution of Some Plane Boundary Value Problems for Perforated Cosserat Elastic Bodies

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Abstract. In the paper we consider some plane boundary value problems of asymmetric theory of elasticity for perforated domains. The domain is the square with holes arranged in the definite manner. The formulated problems are solved approximately by using the method of fundamental solutions is used.

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Key words: Cosserat theory, perforated domains, plane boundary value problems, method of fundamental solutions.

1 Introduction

An elastic body is called perforated if it is weakened by several holes. Perforated elements are widely used in all industrial sectors and everyday life. Hence the calculation of stresses occurring in perforated bodies is an important task. A great number of published scientific works dedicated to this topic also testify to the importance and difficulty of solving the related problems.

In the case of a rigorous formulation of the problem, the calculation of the stability of perforated plates reduces to the consideration of a difficult boundary value problem for multiply connected domains. To cope with the difficulty, much effort was made in the course of many years in order to develop methods which would allow one to replace a real plate by an equivalent simply connected plate of the same dimension. Such a plate was supposed to have the corresponding elastic properties that could be used for a more or less adequate description of the behavior of a real plate. Such methods were developed in the works [1–5].

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With the progress in the development of numerical methods it became possible to construct approximate solutions of boundary value problems for perforated domains [6–9]. For this, two methods were mainly used: the finite element method (FEM) and the boundary element method (BEM). As compared with the FEM, the principal advantage of the BEM is that it exactly satisfies the solution of a system of differential equations inside the considered domain and also makes it unnecessary to split the domain into subdomains (elements). In the present paper we use the fundamental solutions method (MFS) which retains the above-mentioned advantages of the BEM, but, as different from the latter method, removes the need to approximate the boundary. This circumstance allows us to avoid the corresponding errors and not to use local coordinate systems. As a result the process of construction of an approximate solution becomes essentially simpler and this simplification is especially obvious when we consider boundary value problems for multiply connected domains.

It should be noted that of a great number of works on the strength of perforated bodies only a few papers are dedicated to the study of anisotropic plates. As to the study of the strength of perforated elastic bodies on the basis of Cosserat theory [9–14], the number of works in this direction is even less. Numerous studies, including experimental ones [15], confirm the fact that the Cosserat medium, which is described by asymmetric elasticity theory, makes it possible to construct rather exact models of porous, fibrous and cellular materials. Using the Cosserat medium for modeling such materials we can describe the effects (in particular size effects) which are not covered by classical elasticity theory. The Cosserat theory is, as we know, very clear from the standpoint of physics and not much difficult from the standpoint of mathematics. Thus, in our opinion, the solution of boundary value problems for a system of Cosserat equations in the case of perforated domains is of both theoretical and practical interest. This fact served as a stimulus for writing the present paper. Besides we think that the offered method of construction of approximate solutions of boundary value problems is the simplest and most convenient one for the solution of this kind of problems.

2 Two dimensional system of equilibrium equations of cosserat theory and its general solution

Let $Oxyz$ be a Cartesian coordinate system with unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$. Let the cylindrical domain Ω, occupied by a Cosserat elastic medium be in the plane-deformed state which is parallel to the Oxy plane. Then the displacement vector $\mathbf{u}$ and the rotation vector $\mathbf{w}$ have the form

$$\mathbf{u}(x,y)=u(x,y)\mathbf{i}+v(x,y)\mathbf{j}, \qquad \mathbf{w}(x,y)=\omega(x,y)\mathbf{k}.$$

In this case, a homogeneous static system of equilibrium equations is written in terms of displacement and rotation components as [12,14]