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Probability-Guaranteed H_∞-Filtering for 2D-Systems with Random Nonlinearities and Missing Measurements

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Abstract. The probability-guaranteed H_∞-filtering problem for 2D-discrete time systems with random nonlinearities and missing measurements is studied. To characterise the statistical uncertain parameters, the uniform distribution is used. The filter parameters are derived by LMI technique and a special parameter-box is used to guarantee H_∞-performance with pre-specified probability constraints. An example shows the effectiveness of the filtering scheme developed.

AMS subject classifications: 93E11

Key words: Two-dimensional system, probability performance, H_∞-filtering.

1. Introduction

Two-dimensional systems with information propagation in two independent directions have attracted considerable attention due to applications in image data processing and transmission, biomedical imaging processing, multidimensional digital filtering, thermal processes, and water stream heating [2, 6, 10]. In particular, for 2D-systems asymptotic and exponential stability are studied in [24], controller design problems in [4, 16], stability analysis and stabilisation of switched systems in [1, 21], and H_∞-control problems for Markovian jump systems with state-delays and defective mode information in [20]. There are also results concerning the H_∞-filtering design approach — e.g. the problem of H_∞-filtering is solved for uncertain discrete systems [11], for uncertain continuous systems [22], for time delay systems [3, 12] and for stochastic systems [7].

It is worth noting that all the above-mentioned results are based on the assumption that sensor measurements are perfect. However, in practical situations, it is possible that signals are measured during their transmission and some measurements are missing. Moreover,

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the random nonlinearities often occurring in networked environments, cause a particular concern. Therefore, the stability analysis, controller synthesis and filtering design for 1D networked control systems are widely studied [5,9,18]. On the other hand, there are only a few results available for two-dimensional network-based systems — e.g. robust H_∞-filtering with random mixed delays and with intermittent measurements is, respectively, considered in [15] and [14], recursive filtering for nonlinear systems with measurement degradation in [17], and the state estimation in complex networks with randomly occurring nonlinearities and randomly varying sensor delays in [13].

In traditional control theory, it is rarely possible to achieve the prescribed performance of control systems with the probability 1, but it is acceptable to realise the prescribed performance objectives with an expected probability. The probability guaranteed robust H_∞-controller design method was proposed by Yaesh *et al.* [23]. Subsequently, this method found applications in filtering design problems. Hu *et al.* [8] studied the probability-guaranteed H_∞-finite-horizon filtering for nonlinear time-varying systems, Wei *et al.* [19] considered the probability-guaranteed set-membership filtering problems for time-varying systems with incomplete measurements. Nevertheless, to the best of the authors' knowledge, for $2D$-systems the probability-guaranteed H_∞-filtering problems have not yet been studied.

Here, we deal with probability-guaranteed H_∞-filtering problems for discrete-time $2D$-systems with randomly occurring nonlinearities and missing measurements. The main features of this work are:

1. The comprehensive consideration of the systems, including uncertainty parameters, randomly occurring nonlinearities and incomplete measurement information.

2. The development of an H_∞-filter. In particular, we construct a parameter-box, such that H_∞-performance requirement is guaranteed with pre-specified probability constraints.

3. The proof of the stochastic mean-square asymptotical stability of the $2D$-filtering error systems.

2. Problem Formulation

We consider the following uncertain nonlinear discrete-time $2D$-system:

$$\begin{aligned} &\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A(\theta)x(i,j) + \alpha(i,j)g(x(i,j)) + B(\theta)\omega(i,j), \\ &z(i,j) = Mx(i,j), \end{aligned} \tag{2.1}$$

where $\omega(i,j) \in R^{n_\varpi}$ is the disturbance input in $l_2[0,\infty)$, $z(i,j) \in R^{n_z}$ the controlled output, M a known real-valued matrix of appropriate dimension, and

$$x(i,j) := \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \in R^{n_1+n_2}$$