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A Direct Sampling Method for Recovering Electromagnetic Dipolar Sources

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Abstract. An inverse source problem for time-harmonic Maxwell equation is studied and a direct sampling method to recover the location and the moments of electric current dipoles from near- and far-field data at a fixed frequency is developed. The method is based on a new indicator function and requires the evaluation of simple integrals only. It is easily implementable and efficient. Numerical examples demonstrate the robustness of the method with respect to noisy measurements and its capability in identifying the source location and the moments.

AMS subject classifications: 49N45, 65L32, 65N21

Key words: Inverse source problem, Maxwell equation, direct sampling method, dipole, time-harmonic.

1. Introduction

Inverse source problems occur in diverse applications, including the diagnosis of brain abnormalities, the control of prosthetic limbs in medical imaging and finding infinitesimal dipole models for antennas. The main goal of inverse source problems is to determine geometrical and/or physical parameters of a target source such as support, location and profile [2, 9, 12, 13, 17, 25, 28, 30, 32, 36].

In recent decades great efforts have been spent on the development of numerical methods for inverse source problems in acoustic, electromagnetic and elastic wave propagation — cf. [3, 4, 10, 33, 34]. In addition, a variety of numerical methods are devoted to the localisation of the electromagnetic field source problems. Thus, He and Romanov [14] derived an explicit formula for identifying the location and the moment of a single dipole in a bounded homogeneous domain and established a uniqueness result using the magnetic field. Ammari *et al.* [2] obtained a similar result using the tangential components of

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electric or magnetic field and proposed a numerical method to establish the location of a single dipole. Albanese and Monk [1] studied the unique determination of a volume current source with respect to the boundary tangential components of an electromagnetic fields. Inui and Ohnaka [15] proposed a direct boundary integral based identification method for electric dipoles. El Badia and Nara [11] presented an algebraic algorithm to identify the number, the location and the moment of dipolar sources from the tangential components of electric and magnetic fields.

Recently, Zhang *et al.* [35] investigated two-level direct sampling methods to identify multiple multipolar acoustic sources from near-field Cauchy data. The basic idea of introducing indicator functions [35] is initiated by the decaying property of oscillatory integrals. Thus indicators admit local maxima in open neighborhoods of each point source, while the sampling point coincides with the exact source location. This method is then used to recover the locations and intensities of acoustic source points from the far-field data [5]. Such an approach is also connected to the direct sampling methods of the shape identification in inverse scattering problems — cf. Refs. [6,7,16,18–24,29].

The current work is partially motivated by works [5,35]. Here, we are going to develop a direct sampling method for inverse point source problems in vector Maxwell systems. In particular, a non-iterative sampling-type technique to identify the current dipolar sources from near- or far-field measurements is proposed. For this, novel indicator functions based on suitable products and simple integration are used. As the result, the inversion algorithm does not rely on matrix operations and forward solution procedures and no additional regularisation strategy is needed. The method proposed has the following features:

1. It is able to simultaneously identify the location and the moments of the dipolar sources.
2. It is easily implementable and has a low computational cost.
3. It tolerates measurement errors.

The remainder of this paper is organised as follows. Mathematical formulation of direct and inverse problems is given in Section 2. Section 3 introduces a direct sampling method to identify the dipoles from near-field data and considers their determination from far-field data. The stability of the method is studied in Section 4. Numerical experiments are carried out in Section 5 and concluding remarks are in Section 6.

2. Mathematical Formulation

Let us start with notation. In what follows, we respectively use light and bold fonts for scalar and vector terms.

The propagation of electromagnetic waves is considered in vacuum. It is governed by the time-harmonic Maxwell's equations in $\mathbb{R}^3$

$$\nabla \times \boldsymbol{E} - \mathrm{i}k\boldsymbol{H} = 0, \tag{2.1}$$

$$\nabla \times \boldsymbol{H} + \mathrm{i}k\boldsymbol{E} = \boldsymbol{J}, \tag{2.2}$$