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A Dual-Level Method of Fundamental Solutions in Conjunction with Kernel-Independent Fast Multipole Method for Large-Scale Isotropic Heat Conduction Problems

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Abstract. A dual-level method of fundamental solutions in conjunction with kernel-independent fast multipole method is proposed in this study. The competitive attributes of the method are that it inherits high accuracy of the method of fundamental solutions, yet avoids producing the resulting ill-conditioned linear system of equations. In contrast to the method of fundamental solutions, the proposed method places two sets of source nodes on the fictitious boundary and physical boundary, respectively, and then combines the fundamental solutions generated by these two sets of source nodes as the modified fundamental solutions of the Laplace equation. This strategy improves significantly the stability of the method of fundamental solutions. In addition, the method is accelerated by the kernel-independent fast multipole method, which reduces the asymptotic complexity of the method to $\mathcal{O}(N)$ from $\mathcal{O}(N^2)$. Numerical experiments show that the method can simulate successfully the large-scale heat conduction problems via a single laptop with up to 250000 degrees of freedom.

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Key words: Dual-level method of fundamental solutions, isotropic heat conduction problems, ill-conditioning, range restricted GMRES method, kernel-independent fast multipole method.

1 Introduction

The method of fundamental solutions (MFS) [1–5] is an efficient meshless method [6–10] with high accuracy and efficiency. In contrast to the boundary element method

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(BEM) [11–15], the MFS avoids the singularity of the fundamental solutions at origin by placing the source points on the fictitious boundary. Therefore, the time-consuming singular integrations are avoided. In comparison with the singular boundary method (SBM) [16–20], the MFS requires very few degrees of freedom (DOF) to produce highly accurate results.

However, the highly ill-conditioned interpolation matrix is the most important constraint of the MFS for practical engineering computing [21–24]. In recent decades, several techniques have been proposed to reduce the ill-conditioning of the MFS [25–28]. Although this highly ill-conditioned matrix often doesn't affect the quality of the numerical solution, the iterative solver such as the generalized minimal residual algorithm (GMRES) [29] cannot be used to solve the obtained highly ill-conditioned linear system of equations, which restricts directly the application of the MFS for the large-scale engineering problems.

In this study, we propose a dual-level method of fundamental solutions (DLMFS) in conjunction with kernel-independent fast multipole method (IFMM) [30–34] for simulation of the large-scale isotropic heat conduction problems. In the proposed algorithm scheme, the concept of the source points and mirror source points are introduced, which are placed on the fictitious and real physical boundary, respectively. The proposed DLMFS has much lower condition number than the MFS by mixing the fundamental solutions of above two sets of points as the modified fundamental solutions of the Laplace equation. Meanwhile, the accuracy and numerical efficiency of the DLMFS is little affected. In this article, we use the range restricted GMRES iterative solver (RRGMRES) [35,36] which has the regularization function to solve the resulting linear system of equations. In addition, the IFMM is used to expedite the solution process of the DLMFS, which can reduce the asymptotic complexity to $\mathcal{O}(N)$ from $\mathcal{O}(N^2)$.

Numerical experiments with up to 250000 DOF have successfully been achieved for the large-scale isotropic heat conduction problems via a single laptop. And it should be stressed that the MFS cannot simulate the similar problem by using the iterative solver due to its high condition number. A brief outline of this paper is as follows: Section 2 describes the formulations of the DLMFS; Section 3 tests the DLMFS through three benchmark examples; finally, some conclusions and remarks are given in Section 4.

2 Numerical methodology

2.1 Formulations of the dual-level method of fundamental solutions

Consider a 3-D isotropic medium in an open bounded domain Ω bounded by surface Γ. In this study, we refer to steady state isotropic heat conduction applications without inner heat sources. Therefore, the physical variable u, which denotes the temperature