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# An Unconditionally Stable Numerical Method for Two-Dimensional Hyperbolic Equations

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**Abstract.** A collocation method based on exponential B-splines for two-dimensional second-order non-linear hyperbolic equations is studied. The initial equation is split into a system of coupled equations, each of which is transformed into a system of ordinary differential equations. The corresponding differential equations are solved by SSP-RK(2,2) method. It is shown that the method under consideration is unconditionally stable. Numerical experiments demonstrate its efficiency and accuracy.

**AMS subject classifications**: 65M60, 65M06, 65N30, 65N06

**Key words**: Collocation method, SSP-RK(2,2), telegraph equation, tri-diagonal solver, unconditional stability.

## 1. Introduction

We consider the second order two-dimensional non-linear hyperbolic equation

$$u_{tt} = u_{xx} + u_{yy} - 2\alpha u_t - \beta^2 u + g(x,y,t) + f(u), \quad a < x < b, \quad c < y < d, \quad t > 0 \tag{1.1}$$

with the initial conditions

$$u(x,y,0) = \phi(x,y), \quad u_t(x,y,0) = \psi(x,y), \quad a \le x \le b, \quad c \le y \le d \tag{1.2}$$

and the Dirichlet boundary conditions

$$\begin{aligned} u(a,y,t) &= f_1(y,t), \quad u(b,y,t) = f_2(y,t), \quad c < y < d, \quad t > 0, \\ u(x,c,t) &= f_3(x,t), \quad u(x,d,t) = f_4(x,t), \quad a < x < b, \quad t > 0. \end{aligned} \tag{1.3}$$

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If $\alpha > 0$ and $\beta > 0$, the Eq. (1.1) becomes the telegraph equation and it is damped wave equation if $\alpha > 0$ and $\beta = 0$.

This equation is used in diffusion processes [10], image processing [28], vapor phase chromatography [9], dispersal in biological systems [1], and stochastic processes [18, 19]. A considerable attention has been paid to the solution of one-, two- and three-dimensional second order hyperbolic equations. In particular, Mohanty *et al.* [15–17] developed unconditionally stable implicit three level methods for one-dimensional second order hyperbolic problems and unconditionally stable implicit alternating direction methods for two- and three- dimensional hyperbolic problems. For two-dimensional linear telegraph equations, Bülbül *et al.* [4] and Jiwari *et al.* [12] developed Taylor matrix based methods and a differential quadrature method, respectively. Considering two-dimensional second-order hyperbolic equations, Ding and Zhang [7] proposed a fourth-order compact difference scheme, Dehghan and Ghesmati [5] studied meshless local weak and strong form methods and Dehghan and Mohebbi [6] considered a collocation method. In addition, Rashidinia *et al.* [20] and Mittal *et al.* [14] used cubic B-splines in one- and two-dimensional equations, respectively.

Here, we deal with an approximation method based on exponential B-splines. It was shown by McCartin [13] that exponential splines have a number of advantages — i.e. in computational aerodynamics they do not produce false oscillations of interpolants that appear in cubic splines methods. Nevertheless, exponential splines are rarely used in approximate solution of partial differential equations. Thus Ersoy and Idris [8] provided an exponential B-spline based algorithm for the Korteweg-de Vries equation, Singh *et al.* [21] used exponential B-splines in collocation method for one dimensional second order hyperbolic equation. Note that these splines have been introduced by Späth [23], who also considered their two-dimensional generalisation [24]. In this work an exponential B-spline based collocation method is applied to the second order two-dimensional non-linear hyperbolic equation (1.1). Decomposing the Eq. (1.1) into two equations, we discretise them in spatial directions and convert into the systems of ordinary differential equations. The systems obtained, are solved by SSP-RK(2,2) method — cf. Ref. [25].

The paper is organised as follows. In Section 2, we discuss a two-dimensional exponential B-spline based collocation method, with more details being provided in Section 3. Section 4 is concerned with the stability analysis. Five numerical examples are considered in Section 5 and our concluding remarks are in Section 6.

## 2. Two-Dimensional Exponential B-Spline Collocation Method

We consider the partitions

$$a = x_0 < x_1 < \cdots < x_{N-1} < x_N = b, \tag{2.1}$$

$$c = y_0 < y_1 < \cdots < y_{M-1} < y_M = d \tag{2.2}$$

of the domain $\Omega = \{(x, y) : a \le x \le b, c \le y \le d\}$, where $h_x = x_l - x_{l-1} = (b-a)/N$, $l = 1, 2, \cdots, N$ and $h_y = y_m - y_{m-1} = (d-c)/M$, $m = 1, 2, \cdots, M$. Moreover, we use