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# The "Hot Spots" Conjecture on Homogeneous Hierarchical Gaskets

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**Abstract.** In this paper, using spectral decimation, we prove that the "hot spots" conjecture holds on a class of homogeneous hierarchical gaskets introduced by Hambly, i.e., every eigenfunction of the second-smallest eigenvalue of the Neumann Laplacian (introduced by Kigami) attains its maximum and minimum on the boundary.

**Key Words**: Neumann Laplacian, "hot spots" conjecture, homogeneous hierarchical gasket, spectral decimation, analysis on fractals.

**AMS Subject Classifications**: 52B10, 65D18, 68U05, 68U07

## 1 Introduction

The "hot spots" conjecture was posed by J. Rauch at a conference in 1974. Informally speaking, it was stated in [3] as follows: Suppose that $D$ is an open connected bounded subset of $\mathbb{R}^d$ and $u(t,x)$ is the solution of the heat equation in $D$ with the Neumann boundary condition. Then for "most" initial conditions, if $z_t$ is a point at which the function $x \to u(t,x)$ attains its maximum, then the distance from $z_t$ to the boundary of $D$ tends to zero as $t$ tends to $\infty$. In other words, the "hot spots" move towards the boundary. Formally, there are several versions of the hot spots conjecture. See [3] for details. In this paper, we will use the following version: every eigenfunction of the second-smallest eigenvalue of the Neumann Laplacian attains its maximum and minimum on the boundary.

The "hot spots" conjecture holds in many typical domains in Euclidean space, especially for certain convex planar domains and lip domains. For examples, please see [1, 3, 11]. On the other hand, Burdzy and Werner [5] and Burdzy [4] constructed interesting planar domains such that the "hot spots" conjecture fails.

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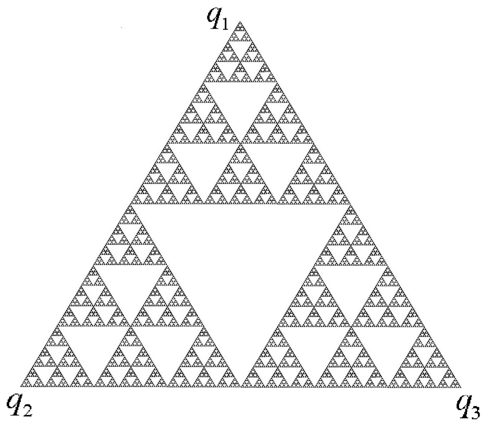


Figure 1: $HH(b)$, where $b=(2,3,3,2,\cdots)$.

The underlying spaces in above works are domains in Euclidean space. Since we can do analysis on fractals (see [12, 13, 21]), it is natural to ask whether the conjecture holds for p.c.f. fractals. Recently, there are some works on this topic. On the one hand, Ruan [17], Ruan and Zheng [18], Li and Ruan [15] proved that the conjecture hold on the Sierpinski gasket ($SG_2$ for short), the level-3 Sierpinski gasket ($SG_3$ for short) and higher dimensional Sierpinski gaskets. On the other hand, Lau, Li and Ruan [14] proved that the conjecture does not hold on the hexagasket. The basic tool used in these paper is spectral decimation.

The above fractals studied are all p.c.f. self-similar. Thus it is interesting to ask whether the conjecture holds for non p.c.f. self-similar fractals. In this paper, we will consider homogeneous hierarchical gaskets, which were introduced by Hambly [8, 9]. These gaskets are non p.c.f.. Fortunately, they admit spectral decimation so that we can use similar method to prove that the conjecture holds on these gaskets.

Roughly speaking, the subdivision scheme for homogeneous hierarchical gaskets is a variant of the one for the usual Sierpinski gasket and constructed level by level. Each cell of level $m$ is contained in a triangle,and that triangle is split into triangles of sides $1/b_{m+1}$ times the side of the original triangle, where $b_{m+1}\in\{2,3,\cdots\}$. If $b_{m+1}=2$, we will have the cell of level $m+1$ as the same construction of $SG_2$, if $b_{m+1}=3$, we will have the cell of level $m+1$ as the same construction as $SG_3$. The resulting gasket is denoted by $HH(b)$ for $b=(b_1,b_2,\cdots)$. In this paper, we will restrict that $b_m$ equals 2 or 3 for each $m$. See Fig. 1 for an example.

Notice that $SG_2$ and $SG_3$ are typical p.c.f. self-similar sets, while generally $HH(b)$ is not a self-similar set. Meanwhile, the Dirichlet Laplacian and the Neumann Laplacian of these gaskets have already been discussed by Drenning and Strichartz [6]. Thus, it is natural to ask whether the hot spots conjecture holds on certain homogeneous hierarchical gaskets.

The rest of the paper is organized as follows. Basic concepts are recalled in Section 2.