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# Coupled MRT Lattice Boltzmann Study of Electrokinetic Mixing of Power-Law Fluids in Microchannels with Heterogeneous Surface Potential

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**Abstract.** The electrokinetic mixing, as a powerful technique in microfluidic devices, is widely used in many applications. In this study, a more general dynamic model, which consists of Poisson equation, Nernst-Planck equation and Navier-Stokes equations, is used to describe the electrokinetic mixing of non-Newtonian fluids in microchannels. Furthermore, a coupled multiple-relaxation-time (MRT) lattice Boltzmann (LB) framework is developed to solve this complicated multi-physics transport phenomenon. In numerical simulations, we mainly consider the effects of the arrangement of nonuniform surface potentials and the power-law index on the mixing efficiency and the volumetric flow rate. Numerical results show that the mixing efficiency and the volumetric flow rate of shear-thinning fluids are higher than that of shear-thickening fluids under the same condition. It is also shown that for both types of fluids, there should be a balance between the mixing and liquid transport in electrokinetic microfluidics.

**AMS subject classifications**: 76A05, 65Z05, 76W05

**Key words**: Lattice Boltzmann method, electrokinetic mixing, non-Newtonian fluids.

## 1 Introduction

The micro-electro-mechanical systems (MEMS) have attracted more and more attentions in recent years due to its wide applications in biomedical, chemical analyses, drug de-

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livery and so on [1–3]. In many practical microfluidic applications, the rheological behavior of the fluids must be considered since the used fluids have the non-Newtonian characteristics. Specifically, non-Newtonian effects are characterized by proper constitutive models which relate the dynamic viscosity and the rate of shear. Power-law fluid model is certainly the most popular because it is simple and able to fit a wide range of non-Newtonian fluids, such as solutions of blood, saliva, protein and DNA, polymeric solutions and colloidal suspensions. One important parameter in the power law fluid model is the fluid behavior index $(n)$ which delineates the dependence of the dynamic viscosity on the rate of shear. If $n$ is smaller (greater) than one, the fluid demonstrates the shear-thinning (shear-thickening) effect that the viscosity of fluid decreases with the increase (decrease) of the rate of shear. If $n$ is equal to one, the fluids then exactly behave as Newtonian fluids. In this study, for the shear-thinning fluids, we consider the supramolecular polypeptide-DNA solutions as the working fluids, while for the shear-thickening fluids, the macromolecular protein solution is considered [4].

In such microfluidic devices, especially in the biomedical applications, rapid and homogenous mixing of two or more fluid species are required. Recently, electroosmotic flow (EOF), as an important non-mechanical actuating technique, has been widely utilized in fluid transport and mixing in microfluidic devices. The flow pattern of EOF in a microchannel is dependent on the surface charge distributions and the external applied electric fields. Existing research results show that the EOF in a microchannel with heterogeneous surface potential induces a vortex flow, which appears to be an excellent alternative to enhance mixing efficiency [5, 6]. This mixing method has also received intensive attention because of the ease of control and integration with microfluidic devices. The EOF is a typical multiphysical transport phenomenon, which involves multiple processes including fluid flow, electrostatic interaction and species diffusion. Generally, this multiphysical transport problem can be described by the Poisson-Boltzmann (PB) model, which consists of the PB equation for the internal electrical potential and the Navier-Stokes equation for the flow field of electrolyte solutions. The PB equation is derived from the assumption that the ionic distribution in the electrical double layer (EDL) is to follow the equilibrium Boltzmann distribution [7, 8]. However, the Boltzmann distribution is only applicable for the cases that the ionic distributions are not affected by the flows of the electrolyte solution or the bulk solution is far away from the charged surface. Thus, a more general model, could also be called Nernst-Planck (N-P) model, which utilizes the N-P equation for ions transport instead of Boltzmann distribution should be adopted in the study of EOF in microchannels with heterogeneous surface potential [9–12]. In the N-P model, the velocity of the flow field, the electrostatic potential and the ionic concentration are coupled together with strong nonlinearity, which poses great challenge to the numerical solution. In the past few years, the lattice Boltzmann method (LBM), which originates from lattice gas automata (LGA) and also could be derived from the kinetic Boltzmann equation, has emerged as an alternative powerful numerical method for simulating complex flows [13–15] and also has been extended to solve convection-diffusion-type equations [16–19]. In contrast to the classical PDE solvers, such as the