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The New Mode of Instability in Viscous High-Speed Boundary Layer Flows

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Abstract. The new mode of instability found by Tunney et al. [24] is studied with viscous stability theory in this article. When the high-speed boundary layer is subject to certain values of favorable pressure gradient and wall heating, a new mode becomes unstable due to the appearance of the streamwise velocity overshoot ($U(y) > U_\infty$) in the base flow. The present study shows that under practical Reynolds numbers, the new mode can hardly co-exist with the conventional first mode and Mack's second mode. Due to the requirement for additional wall heating, the new mode may only lead to laminar-turbulent transition under experimental (artificial) conditions.

AMS subject classifications: 65M10, 78A48

Key words: Boundary layer stability, compressible boundary layers, high-speed flow.

1 Introduction

The mechanism of high-speed laminar-turbulent flow transition is far from fully understood [16]. One important reason is the multitudinous routes of the transition process that is in turn influenced by various environmental conditions. Among them, modal stability is generally considered the fundamental mechanism and relatively well-studied. The representative examples are Tollmien-Schlichting waves in (quasi-) parallel flows [26], Mack's second modes in hypersonic flows [2], cross-flow modes in three-dimensional boundary layers [13] and Görtler modes over concave surfaces (when Reynolds number is large) [8]. Under certain conditions (particularly with low external turbulence and smooth geometry), perturbations (generated through receptivity mechanism) get amplified with modal instabilities causing the flow close to transition when their amplitude

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becomes large. However, even after amounts of studies, the knowledge on this fundamental modal stability is still insufficient.

Compared with zero pressure gradient, favorable pressure gradient (hereafter referred to as FPG) significantly stabilizes the boundary layer in both incompressible and compressible flows (the first mode as well as Mack's second mode). This is supported by a number of studies, e.g., with direct numerical simulation [5, 10, 18], linear stability theory [4,6,7] and very recent experiments [23,25]. Hence, in the review by Reed et al. [9], the instability of boundary layer with FPG is described as "very weak, if it exists at all". In fact, with FPG, the profile of the base flow $U(y)$ becomes fuller and the thickness of the boundary layer is decreased, which is mainly responsible for the stabilization of the boundary layer.

On the other hand, wall-heating/cooling is one of the common passive flow control methods used on various occasions. Its influence on boundary layer stability has been well documented (see reviews in [3, 9]). In contrast to the adiabatic condition, wall heating can destabilize the first mode while stabilizing Mack's second mode. Wall cooling, instead, has opposite effects. One shall distinguish between wall-heating and localized wall-heating. The latter gives rise to wall temperature jump effect and can destabilize Mack's second mode (see recent analysis in [22]).

When the flow is subject to the dual effects of FPG and wall-heating, a new mode comes to light. A first analytical study was performed by Tunney et al. [24] under the inviscid assumption. The direct cause of the instability is the appearance of streamwise velocity *overshoot* ($U(y) > U_\infty$ near the upper edge of the boundary layer). Discussion on the *overshoot* can be found in Tunney et al. [24] and the references therein. Under inviscid assumption, the new mode was shown to have comparable growth rate as the conventional first mode and Mack's higher mode. However, the possible importance of the new mode is not evaluated. In this paper, we report a viscous stability analysis (with spatial mode) on this new mode which is more relevant for developing boundary layers. The impact and limitations of the new mode will be discussed. In Section 2 the methodology and the base flow are introduced. Modal stability is discussed in Section 3 and the paper is concluded in Section 4.

2 Methodology and base flow

The stability equations are derived from the Navier-Stokes equations provided the base flow is obtained in advance. A frequently-adopted form is written as

$$\begin{aligned}&\boldsymbol{\Gamma}\frac{\partial\tilde{q}}{\partial t}+\boldsymbol{A}\frac{\partial\tilde{q}}{\partial x}+\boldsymbol{B}\frac{\partial\tilde{q}}{\partial y}+\boldsymbol{C}\frac{\partial\tilde{q}}{\partial z}+\boldsymbol{D}\tilde{q}\\ =&\boldsymbol{V}_{xx}\frac{\partial^2\tilde{q}}{\partial x^2}+\boldsymbol{V}_{xy}\frac{\partial^2\tilde{q}}{\partial x\partial y}+\boldsymbol{V}_{xz}\frac{\partial^2\tilde{q}}{\partial x\partial z}+\boldsymbol{V}_{yy}\frac{\partial^2\tilde{q}}{\partial y^2}+\boldsymbol{V}_{yz}\frac{\partial^2\tilde{q}}{\partial y\partial z}+\boldsymbol{V}_{zz}\frac{\partial^2\tilde{q}}{\partial z^2}.\end{aligned} \tag{2.1}$$