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Mortar Element Method for the Coupling of Navier-Stokes and Darcy Flows

Xin Zhao[1] and Yanping Chen[2,*]

[1] *School of Mathematics and Computational Science, Xiangtan University, Xiangtan 411105, China*
[2] *School of Mathematical Sciences, South China Normal University, Guangzhou 510631, China*

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Abstract. In this paper, we investigate the nonlinear system of Navier-Stokes and Darcy flows coupled by the Beavers-Joseph-Saffman law on the interface. The well-posedness of the weak and numerical solutions are established. Mortar element method is introduced to solve the coupled model. Error estimates are derived for the proposed method. We adopt an iterative algorithm to solve the nonlinear scheme generated by the coupled system. Numerical example is provided to verify the theoretical analysis.

AMS subject classifications: 76D05, 76S05, 65N30

Key words: Darcy flow, Navier-Stokes equations, finite element method, mortar element.

1 Introduction

The coupling of Navier-Stokes and Darcy flows has become a topic of significant interests in recent years. In this model, the domain is formed by a porous media and a region in which there is an incompressible fluid. The coupled model requires to consider different systems of partial differential equations in each subregion. The behaviour of the fluid is governed by Navier-Stokes equations. And the porous media satisfies Darcy law. The interface conditions are given by mass conservation, the balance of the normal forces and the Beavers-Joseph-Saffman law. The Beavers-Joseph [5] condition was derived experimentally. And it was simplified by Saffman [6]. The Beavers-Joseph-Saffman law states that the friction coefficient of a slip for Navier-Stokes flow along the interface is related to the permeability of porous media. These coupled models have interesting applications.

*Corresponding author.
Emails: yanpingchen@scnu.edu.cn (Y. P. Chen)

They can be used to simulate the effect of flooding in dry areas, to predict the propagation of pollution in the rivers (lakes) making its way to groundwater and water supply. Meanwhile, the coupled system can model the filtration of blood through arterial vessel wall.

Omitting the nonlinear convection term from Navier-Stokes equations leads to the coupling of Stokes and the Darcy flows. This linear coupled system has been extensively studied. According to the way treating Darcy equations, the methods for the coupled system fall into two categories.

One treats the Darcy equations as a Poisson problem, such as [7–14]. In this formulation, the velocity is obtained by taking the gradient of pressure and multiplying it by the rock permeability tensor. In [7], Discacciati et al. showed the well-posedness of the coupled problem. Discacciati, Quarteroni and Valli [8] presented domain decomposition methods for the coupled system. To decouple the coupling of Stokes and Darcy flows, Mu and Xu proposed a two-grid method in [9]. Cai and Mu [10] generalized the algorithm of [9] to a multilevel algorithm. Zuo and Hou [11] analyzed a modified two-grid method. Besides, in the articles [13, 14], Cao et al. investigated Stokes-Darcy flows coupled by the Beavers-Joseph condition.

The other is based on the mixed form, such as [19–29]. It is a more popular approach in applications. Many well-known finite element spaces can be used to approximate this mixed formulation such as: the Raviart-Thomas spaces [15], the BDM spaces [16], the BDFM spaces [17] and so on. These discrete spaces have been used successfully in numerous applications. Good accuracy is obtained for both velocity and pressure and mass conservation is achieved.

Layton and the coworkers [19] established the Ladyzhenskaya-Babuška-Brezzi (LBB) condition of the weak solution. They gave an analysis of finite element scheme which allows to simulate the coupled problem. Layton et al. introduced a Lagrange multiplier to impose the continuity of the velocity. Afterwards, based on the formulation developed in [19], scholars and researchers have constructed a lot of scheme for different applications. A locally conservative method for the coupled system is analyzed in [20]. Vassilev and Yotov [21] studied the coupled system with an advection-diffusion equation that can model the transport of a chemical. In [22], Rui and Zhang studied the nonconforming finite element methods for the coupled model. And the nonconforming method is proposed for the model with transport by Rui and Zhang [25]. In the paper [23], Babuška and Gatica developed a reliable and efficient residual-based a posteriori error estimator for the coupled model.

Bernardi and the coworkers [27] combined the mortar element method—a domain decomposition technique introduced in [18] with finite element method to discretize this problem. In [28], Girault et al. investigated coupled Stokes and Darcy flows on the multi-domain of either Stokes or Darcy type. They chose coarse scale mortar element on the interface and fine scale finite elements in each subdomain. Recently, Huang, Chen and Cai have combined mortar method with nonconforming elements to discretize the coupled problem in [29].