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Adjoint-Based Adaptive Isogeometric Discontinuous Galerkin Method for Euler Equations

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Abstract. In this work, an adjoint-based adaptive isogeometric discontinuous Galerkin method is developed for Euler equations. Firstly, the solution space used in DG on each cell is constructed with the locally owned geometric representation by the isogeometric concept. Then the local h-refinement is applied directly through Bézier decomposition, without the restrictions of tensor product nature or basis function support. Furthermore, the adjoint-based error estimator is employed to enhance the estimation of practical engineering outputs. With the isogeometric concept, a novel and natural adjoint space is proposed for the associated discrete adjoint problem. Several numerical examples are selected to demonstrate its ability of handling curved geometry, capturing shocks as well as efficiency in reducing the computational cost in comparison to uniform mesh refinement.

AMS subject classifications: 65M60, 76M10

Key words: Discontinuous Galerkin, isogeometric analysis, adjoint-based error estimation, adaptive refinement, Euler equations.

1 Introduction

With the rapid development of computer technology, high-order methods have attracted significant interest in the Computational Fluid Dynamics (CFD) community due to theirs considerable potential [1–3]. Besides, adaptive mesh refinement (AMR) is combined more frequently with these methods than ever due to its efficiency in reducing the computational cost in comparison to uniform mesh refinement [4–6]. However, even

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well-refined linear mesh may lead to inaccurate numerical solution [7], which indicates that high order mesh, or exact geometry, is necessary in practical engineering applications.

As an attractive approach in the Computer Aided Design (CAD) and Finite Element Methods (FEM) communities, isogeometric analysis (IGA) can handle the issue of generating new mesh points naturally because of its intrinsic possession of exact geometry [8, 9]. Furthermore, as a framework for solving PDEs, IGA aims at eliminating or significantly reducing the computational cost for mesh generation, by expanding the solution space with the same basis as that of the geometry description, such as Non-uniform Rational B-splines (NURBS), T-Splines etc. [10, 11]. Because of its positive attributes, the interest in IGA has grown very rapidly since it's born, with applications to many complicated engineering problems [8, 12, 13]. Various adaptive isogeometric methods have also been proposed to extend the application of IGA, such as hierarchical splines, T-splines, as well as locally refined B-splines, etc. [14–16]. A detailed computational comparison among these techniques was presented recently [17]. However, h-refinement in IGA may cause some issues on hanging nodes or linear independence without specified restrictions.

Besides, from a point of solving CFD problems, IGA is not very suitable for these problems governed by compressible Euler equations because the high-order continuity of the basis functions is not consistent with the presence of discontinuities in the solution [18], while discontinuous Galerkin (DG) methods are more popular in CFD community. Furthermore, DG can handle hanging nodes intuitively, which would not suffer from the issues that may arise in adaptive isogeometric methods. As a consequence, in this work, to combine the advantages in mesh generation and points insertion of IGA and the freedom in h-refinement of DG, we propose an isogeometric DG method combined with adjoint-based error estimation and mesh adaption framework for local h-refinement on the basis of our previous work [19]. The model equations are solved under DG framework with grid cells being the Bézier decomposition of the entire physical domain represented by NURBS. Since each cell has its own independent Bézier representation, the isogeometric concept can be applied to each cell individually, and the local h-refinement is also applicable. Furthermore, the adjoint-based error estimator is employed to enhance the estimation of practical engineering outputs. By making use of the features of isogeometric concept, we solve the adjoint problem in a novel space enriched by knot insertion. This space is different from those spaces obtained by h, p, or r-refinement frequently used in other adjoint-based adaptive DG references [20]. The proposed adjoint space is unique to our isogeometric DG method. We also compare the results between this adjoint space and the p-refined space.

The rest of this article is organized as follows. Our isogeometric DG method is reviewed in Section 2, including a brief introduction to IGA and Bézier decomposition. The method of adjoint-based error estimator for the isogeometric DG method is introduced in Section 3. Then some typical numerical examples are illustrated in Section 4. And we come to a conclusion in Section 5.