Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **10**, No. 2, pp. 485-516

DOI: 10.4208/aamm.OA-2017-0125
April 2018

Multiscale Buckling and Post-Buckling Analysis: a Comprehensive Review

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Received 8 May 2017, Accepted (in revised version) 18 October 2017

Abstract. Buckling and post-buckling have been expensively considered as critical limit states of slender/thin members over the centuries. Recent research interests have been shifted to exploit the potential "smart applications" of buckling-induced elastic instability at multiscale. For example, buckling elements have been implemented to develop monostable, bistable and multistable mechanisms for different functional utilities. To design buckling-induced techniques and optimize outputs, it is of necessity to accurately control the mechanical response of buckling elements. This review paper aims at showcasing the theoretical studies of multiscale buckling and post-buckling analysis mainly conducted during the last two decades. The studies of micro/nanoscale buckling and post-buckling analysis are discussed with respect to the nonlocal elasticity, non-classical couple stress and strain gradient elasticity theories, respectively, while the macroscale buckling analysis is categorized into the small and large deformation studies. A comparison is presented between the small and large deformation models at macroscale to identify the applicability of the theories. Recent research trends are reviewed on the optimal design of buckling-induced mechanisms and techniques. In the end, we discuss the potential research avenues for future innovations.

AMS subject classifications: 74B20, 74G60, 74K35

Key words: Buckling and post-buckling analysis, micro/nanoscale, macroscale, theoretical modeling, smart applications.

1 Introduction

Buckling and post-buckling have been expensively considered as critical failure limit states over the centuries. Research efforts are dedicated to investigating buckling-induced

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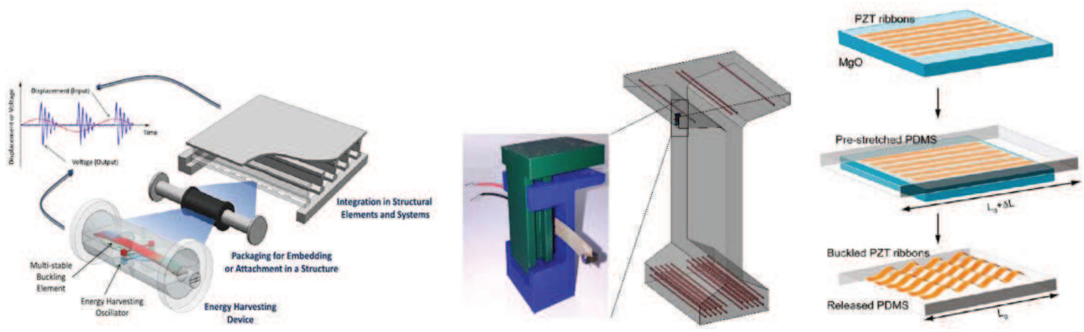

(a) (b) (c)

Figure 1: Buckling-induced mechanisms and "smart applications". (a) Schematic illustrating an energy harvester using a bilaterally constrained macrobeam subjected to axial loading [12]. (b) Applying a buckling-enable device to detect limit states under thermal load [150]. (c) An energy harvesting device using buckled Lead Zirconate Titanate (PZT) ribbons [162].

mechanical response, namely structural instability, while preventing it from happening [1–4]. In recent years, research interests have been shifted to exploit the potential "smart applications" of buckling-induced elastic instability at multiscale [5–7, 159, 160]. Buckling elements have been implemented to develop monostable, bistable and multistable mechanisms for different applications, e.g., actuation, MEMS switch, damage sensing, energy harvesting, etc. [8–11, 161, 192]. For example, buckling-based mechanisms have been proposed to transform ambient energies, namely strain or thermal energies, into electrical power under low frequency excitations (<1Hz). The electrical power/signal can be used to charge remote wireless sensors [12, 13, 162] and detect limit states of structures subjected to thermal load [150]. Thanks to the energy harvester, wireless sensors are deployed in civil infrastructures for structural health monitoring (SHM) [14–20, 151, 152, 157]. Fig. 1 presents typically buckling-induced mechanisms and applications. To design the techniques and optimize outputs, it is of necessity to accurately predict and control the buckling behavior of slender/thin elements. Many studies have been conducted to theoretically investigate buckling and post-buckling response at multiscale [145].

This paper aims at summarizing and characterizing the theoretical studies of buckling and post-buckling analysis mainly published during the last two decades, comparing the models at multiscale to identify the applicability. Section 2 presents an overview of the research trends on multiscale buckling and post-buckling analysis. Section 3 discusses the typical theories developed on buckling and post-buckling analysis of slender/thin members at micro/nanoscale, i.e., the nonlocal elasticity theory, non-classical couple stress theory and strain gradient elasticity theory. Section 4 reviews the theories at macroscale in terms of small and large deformation assumptions. Particular discussions are provided regarding irregular, i.e., the geometric properties of buckling elements are non-uniform,