Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **10**, No. 2, pp. 409-423

DOI: 10.4208/aamm.OA-2017-0011
April 2018

A Novel Discretization Method for Semilinear Reaction-Diffusion Equation

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Received 12 January 2017, Accepted (in revised version) 24 July 2017

Abstract. In this work, we investigate a novel two-level discretization method for semilinear reaction-diffusion equations. Motivated by the two-grid method for nonlinear partial differential equations (PDEs) introduced by Xu [18] on physical space, our discretization method uses a two-grid finite element discretization method for semilinear partial differential equations on physical space and a two-level finite difference method for the corresponding time space. Specifically, we solve a semilinear equations on a coarse mesh $\mathcal{T}_H(\Omega)$ (partition of domain Ω with mesh size H) with a large time step size Θ and a linearized equations on a fine mesh $\mathcal{T}_h(\Omega)$ (partition of domain Ω with mesh size h) using smaller time step size θ. Both theoretical and numerical results show that when $h=H^2,\theta=\Theta^2$, the novel two-grid numerical solution achieves the same approximate accuracy as that for the original seminlinear problem directly by finite element method with $\mathcal{T}_h(\Omega)$ and θ.

AMS subject classifications: 65M10, 78A48

Key words: Two-level discretization method, semilinear reaction-diffusion equation, convergence analysis.

1 Introduction

Two-grid discretization method, which was first proposed by Xu [18, 19] for physical space by finite element methods, has become a very popular method in dealing with the semilinear/nonlinear problems in numerical simulation. The main idea of the two-grid method is based on the observation that a very coarse grid is sufficient for some nonsymmetric, indefinite and/or nonlinear problems that are dominated by their symmetric,

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positive and/or linear parts. In this paper, we shall extend this method on both physical and time space for semilinear parabolic equations.

Two-grid discretization method has been widely used for different kinds of problems, such as elliptic equations [18,19], parabolic equations [2,7,8,11,14], hyperbolic equations [1,3], eigenvalue problems [13,20,22,23], stochastic partial differential equations [6], fractional differential equations [15,16] and so on. The technique is also combined with different kinds of existed numerical methods for discretizing continuous partial differential equations [4,6,10,12,18], for instance, two-grid finite difference method [12], two-grid finite element method [18,19], two-grid finite volume method [1,4], two-grid discontinuous Galerkin method [10,21] and two-grid spectral collocation method [6]. The purpose of this study is to improve the efficiency of the finite difference-finite element method for time dependent semilinear PDEs. Our motivation comes from study [6] in which a novel two-level discretization method is used for the partial differential equations with random input data. In order to deal with the nonlinear property on both physical and random space, the study [6] employed the two-level stochastic collocation points for the random space and two-grid finite element method for the physical space. This idea can also be extended for nonlinear parabolic problems in which there are also two different spaces i.e., the time space and the physical space.

Two-grid discretization technique based on finite difference or mixed finite element method have been studied for nonlinear parabolic equations such as [2,7,8,11,14]. In these studies, the two-grid technique is only used for physical spaces. In this paper, we will concern applying the idea of two-level technique on both physical space and time space (i.e., two-grid finite element methods for physical space and two-grid finite difference method for time space). More specifically, we will solve a semilinear equations on the coarse mesh $\mathcal{T}_H$ with large time step size Θ and linearized equation on fine mesh $\mathcal{T}_h$ with small time step size θ. We show that the two-grid approximate solution achieves the same convergence accuracy when the coarse mesh size and fine mesh size satisfy $h=H^2$ and $\theta=\Theta^2$. Numerical experiments are given to verify the results.

The outline of the paper is as follows. In Section 2, we introduce the model problem and some notations. The two-level method is described in detail in Section 3. In Section 4, we estimate the error of the approximated solution. Finally, in Section 5, two simple numerical examples are given to verify the theoretical results.

2 Model problem and weak formulation

In this paper, we consider the following semilinear parabolic equations

$$u_t-\Delta u=f(u), \quad (x,t)\in\Omega\times\Gamma, \tag{2.1}$$

with boundary and initial conditions

$$u|_{\partial\Omega}=0, \quad u|_{t=0}=u_0,$$