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# Transition Flow with an Incompressible Lattice Boltzmann Method

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**Abstract.** Direct numerical simulations of the transition process from steady laminar to chaotic flow are considered in this study with the relatively new incompressible lattice Boltzmann equation. Numerically, a multiple relaxation time fully incompressible lattice Boltzmann equation is implemented in a 2D driven cavity. Spatial discretization is 2nd-order accurate, and the Kolmogorov length scale estimation based on Reynolds number (*Re*) dictates grid resolution. Initial simulations show the method to be accurate for steady laminar flows, while higher *Re* simulations reveal periodic flow behavior consistent with an initial Hopf bifurcation at *Re* 7,988. Non-repeating flow behavior is observed in the phase space trajectories above *Re* 13,063, and is evidence of the transition to a chaotic flow regime. Finally, flows at Reynolds numbers above the chaotic transition point are simulated and found with statistical properties in good agreement with literature.

**AMS subject classifications**: 65M10, 78A48

**Key words**: Multiple relaxation time, lattice Boltzmann, transition, high Reynolds number flow, incompressible flow, lid driven cavity.

## 1 Introduction

Characteristics of flow transition from steady state to transience and from transience to self-sustaining chaos and instability is an active area of research. While transition to chaos (turbulence in 3D) and separation remain unsolved problems in engineering flow analysis, direct numerical simulation (DNS) work continues to provide insight to the physics in an effort to develop improved turbulence transition and separation models. Past application of the lattice Boltzmann method to chaos and turbulence simulation, such as channel flow by Lammers et al., suffered from the compressibility error [11, 15] inherent in the standard lattice Boltzmann equation [8, 20]. This work extends the study

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of transitional flow states using an incompressible lattice Boltzmann method (iLBM) and a Multiple Relaxation Time (MRT) collision operator. Although physical turbulence is a three dimensional phenomenon, solutions to the flow equations in two dimensions can exhibit similar randomness.

Bifurcations, chaos, and energy dissipation, among other characteristics, exist, while vortex stretching is absent. With this in mind, this work provides a numerical solution benchmark using MRT-iLBM for comparison with past 2D work.

The Lattice Boltzmann Method (LBM) is a relative newcomer in the field of Computational Fluid Dynamics (CFD), having been first described in 1988 [16]. This class of analysis methodology can take many forms. For continuum flow analysis, parameters are chosen such that numerical solutions to the Navier-Stokes equations are recovered through solving the discrete Boltzmann equation. While the behavior of Navier-Stokes is replicated by LBM, the underlying solver algorithms for LBM are considerably simpler than an equivalent Navier-Stokes solver. Reduced computations-per step reduce overall roundoff error and make the MRT-iLBM a less numerically noisy solution method [10].

Lid-driven cavity flow (LDC) is a canonical flow case useful in evaluating methodology [4]. Literature reporting results for steady and transient laminar, transitional, and chaotic flows using a wide variety of methods is extensive. The complexity of the flow despite simple boundaries, along with the plethora of results, makes it an excellent verification tool. Steady state results were obtained by Ghia et al. [6] using a vorticity-stream function approach up to moderate Reynolds numbers. More recently, fine-grid results were reported by Marchi et al. [13] using finite volume Navier-Stokes. LBM has been applied to the LDC flow in the past by Hou et al. [9], who point out the compressibility error present in their results.

Past studies of 2D driven cavity flow have reported various values for the Reynolds number at the onset of transient behavior, dependent on the balance of noisiness and dissipation of the numerical method used. Cazemier et al. [5] analyzed this flow using both DNS and a reduced-order model deduced from Proper Orthogonal Decomposition of a DNS simulation performed at *Re* 22,000. Peng et al. [18] reported the Reynolds numbers of the first Hopf bifurcation and the turbulence transition, but the sixth order numerical method applied in the domain center was contaminated by larger error terms from the second order spatial scheme applied at the boundaries. It is possible that numerical noise from the combination of low-order accurate numerics and typical grid resolution at the walls - where the highest gradients occur - served as a source of artificial excitation and contributed error to the determination of transitional *Re*. Marie et al. [14] show the dispersion and diffusion error of LBM to be excellent across all wavenumbers, suggesting it is well-suited for DNS analysis.

Several authors report steady solutions of the 2D lid driven cavity flow at Reynolds numbers upwards of 30,000 by omitting time dependency terms in the modeled equations. Failure of these simulations to allow solution unsteadiness belies the fact that the existence of steady flow depends upon Reynolds number-dependent stability criteria. Solution unsteadiness below *Re* 10,000 has been well established since at least the work