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A Novel Perturbative Iteration Algorithm for Effective and Efficient Solution of Frequency-Dependent Eigenvalue Problems

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Abstract. Many engineering structures exhibit frequency dependent characteristics and analyses of these structures lead to frequency dependent eigenvalue problems. This paper presents a novel perturbative iteration (PI) algorithm which can be used to effectively and efficiently solve frequency dependent eigenvalue problems of general frequency dependent systems. Mathematical formulations of the proposed method are developed and based on these formulations, a computer algorithm is devised. Extensive numerical case examples are given to demonstrate the practicality of the proposed method. When all modes are included, the method is exact and when only a subset of modes are used, very accurate results are obtained.

AMS subject classifications: 34L15, 34L16, 35P15, 35P30

Key words: Frequency dependent eigenvalue problem, structural vibration, computer algorithm, damped structural vibration, perturbative iteration, eigenvalue and eigenvector sensitivity.

1 Introduction

Frequency dependent systems are those whose system characteristics change with externally applied stimulating frequency. Many natural practical systems are found to be substantially frequency dependent. These include mechanical engine mounts [1], rotor systems [2], viscoelastic and composite structures [3,4], human pelvis systems [5], electrical inductances [6], magnetic modulation systems [7] etc. When such systems are to be analyzed and identified to establish their dynamic characteristics, frequency

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dependent eigenvalue problems are often encountered which need to be solved. A frequency depenedent eigenvalue problem can be defined as

$$\mathbf{A}(\lambda)\boldsymbol{\varphi} = \lambda\mathbf{B}(\lambda)\boldsymbol{\varphi}, \tag{1.1}$$

where **A** and **B** are system matrices which can be complex in general but symmetric and which are functions of eigenvalue (frequency) λ while its conventional counterpart is written as

$$\mathbf{A}\boldsymbol{\varphi} = \lambda\mathbf{B}\boldsymbol{\varphi}. \tag{1.2}$$

Eigenvalue problem is a major branch of applied mathematics and has found wide range of applications [8]. Clasical eigenvalue problems are involved in many engineering disciplines and development of effective and efficient eigensolution methods has remained to be very active during the last few decades. To establish a complete set of eigenvalues and eigenvectors, various algorithms have been developed such as Jocobi method [9], QR method [10] and the more general QZ method [11]. While in the case where only few of the eigenvalues and eigenvectors are of interest, inverse iteration method [12] has been developed and can be effectively employed.

For large practical engineering systems, Lanczos method [13] was probably the first method which was successfully applied to solve large scale eigenvalue problems. Lanczos introduced a recursive algorithm based on a Krylov sequence to determine a subset of eigensolutions. Based on similar concept, Arnoldi [14] presented an effective method for large eigenvalue problems. Based on the formulation of an orthogonal set of vectors, an effective algorithm called Ritz method was developed as discussed in [15]. Alongside with these methods, subspace iteration technique, which has been proven to be very efficient and effective, especially for large practical systems, was introduced [16–18]. Nevertheless, these methods were initially developed for undamped systems, when system damping is considered and hence the eigenvalue problem becomes complex in nature, these methods need to be further extended. Complex Lanczos method was discussed intensely in engineering science in 1980's [19,20]. Arnoldi's method was further developed by Saad [21] to tackle eigenvalue problems of damped systems. A generalized Ritz algorithm for quadratic eigenproblems was presented by Zheng et al. [22].

For conventional eigenvalue problems, extensive research has been conducted and many effective and efficient methods have been developed to date to solve them as the literature review above has indicated. However, frequency dependent eigenvalue problem has not been properly investigated despite its practical importance and there has yet to exist an effective eigensolution method to address this type of eigenvalue problems. This paper seeks to develop a novel perturbative iteration (PI) algorithm which can be used to effectively and efficiently solve frequency dependent eigenvalue problems of general frequency dependent systems. Mathematical formulations of the proposed method are developed and based on these formulations, a numerical algorithm is devised. Extensive numerical case examples are given to demonstrate the