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Stability and Unstability of the Standing Wave to Euler Equations

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Abstract. In this paper, we first discuss the well-posedness of linearizing equations, and then study the stability and unstability of the 3-D compressible Euler Equation, by analysing the existence of saddle point. In addition, we give the existence of local solutions of the compressible Euler equation.

AMS subject classifications: 35Q31, 35L04, 35C07, 35B35

Key words: Euler Equations, Lie Group, well-posedness, saddle point.

1 Introduction

In this paper we consider the compressible Euler system

$$\begin{cases} \partial_t\rho+\nabla_x\cdot(\rho v)=0, \\ \partial_t(\rho v)+\nabla_x\cdot(\rho v\otimes v)+\nabla_x p=0, \\ \partial_t\Big[\rho\Big(e+\dfrac{1}{2}|v|^2\Big)\Big]+\nabla_x\cdot\Big[\rho v\Big(e+\dfrac{1}{2}|v|^2\Big)\Big]+\nabla_x\cdot(pv)=0, \end{cases} \tag{1.1}$$

with the equation of state

$$p=\rho RT=\frac{2}{3}\rho e, \tag{1.2}$$

where (ρ,v,T) represent the macroscopic density, velocity, temperature, respectively. Compressible Euler system (1.1) is a fundamental example of a system of hyperbolic conservation laws. The three equations in the compressible Euler system (1.1) express

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respectively conservation of mass, momentum and energy. It is well-known [1] that the system (1.1) is strictly hyperbolic if the density is bounded below from zero: $\rho > 0$.

Althought the study of the compressible Euler Equation has a long history, stability still be one open problem. Recalling that, namely when the system is strictly hyperbolic everywhere, one can use the theory of symmetric hyperbolic systems developed by Friedrichs-Lax-Kato to construct smooth solutions; for instance, see Majda [2]. The breakdown of classical solutions was demonstrated by Sideris [3]. Sideris [3] gave a sufficient conditions for nonglobal existence of C^1 solutions when $\rho^0(x) > 0$, where $\rho^0(x) > 0$ is the initial density. The nonexistence of C^1 solutions in [3] is related to shock formation. When $\inf \rho^0(x) = 0$, Makino [4] proved the nonglobal existence of regular solutions by assuming the initial data $(\rho^0(x), v^0(x))$ to have compact support, where $v^0(x)$ is the initial velocity. In [5], the authors discovered the Lax pairs of the two-dimensional and three-dimensional Euler equations. In [6], the authors excluded the possibility of δ function or bigger blowup for spherically symmetric or non-spherically symmetric gas by the total potential energy control and Fourier transformation. Li [7] obtained two types of special solutions to Euler equations with spherical symmetry. The authors in [8] present some explicit self-similar blow-up solutions and some other solutions of the incompressible three-dimensional Navier-Stokes equations. These solutions indicate that in C^∞ the solution of Navier-Stokes equations does not always tend to a solution of Euler equations.

The purpose of this article is to establish the stability and unstability of the standing Wave to the compressible Euler equations. After linearization about the uniform equilibrium $\rho = T = 1$, $v = 0$ of the compressible Euler system and a suitable choice of units, the fluid fluctuations (σ, u, θ) satisfy

$$\begin{cases} \partial_t \sigma + \nabla_x \cdot u = 0, \\ \partial_t u + \nabla_x (\sigma + \theta) = 0, \\ \partial_t \theta + \dfrac{2}{3} \nabla_x \cdot u = 0. \end{cases} \tag{1.3}$$

For discussing the stability and unstability of (1.1), we first study the stability of the linearizing equations (1.3): referring to the ideas of reference [9], by using the Lie Group method, we get some explicit solutions of (1.3), then we give the proof that the well-posedness of those explicit solutions. Therefor we prove the stability and unstability of the 3-D compressible Euler Equation, by analysing the existence of saddle point.

Besides the method of this paper, there are other methods to study nonlinear problems, such as [10–14]. Xing Lü in [10,12], derive bright and dark envelope solutions for a generalized mixed nonlinear Schrödinger models, by virtue of the corresponding solitary wave solutions for the generalized stationary Gardner equations. Xing Lü, in [11], solve the three coupled higher order nonlinear Schrödinger equations with the achievement of N-soliton solution formula, by means of Darboux transformation. Xing Lü, in [11], directly construct a bilinear Bäcklund transformation of a (2+1)-dimensional Korteweg-de Vries-like model, based on a so-called quadrilinear representation. In [11], the Gerdjikov-CIvanov envelope solitons are derived and discussed, under suitable hypothesis for the