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Nonlinear Vibration Analysis of Functionally Graded Nanobeam Using Homotopy Perturbation Method

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Abstract. In this paper, He's homotopy perturbation method is utilized to obtain the analytical solution for the nonlinear natural frequency of functionally graded nanobeam. The functionally graded nanobeam is modeled using the Eringen's nonlocal elasticity theory based on Euler-Bernoulli beam theory with von Karman nonlinearity relation. The boundary conditions of problem are considered with both sides simply supported and simply supported-clamped. The Galerkin's method is utilized to decrease the nonlinear partial differential equation to a nonlinear second-order ordinary differential equation. Based on numerical results, homotopy perturbation method convergence is illustrated. According to obtained results, it is seen that the second term of the homotopy perturbation method gives extremely precise solution.

AMS subject classifications: 74G10, 74H45, 74E30,74B99

Key words: Homotopy perturbation method, Lindstedt-Poincare method, analytical solution, nonlocal nonlinear free vibration, functionally graded nanobeam.

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1 Introduction

Functionally graded (FG) materials are a new class of composite materials. These composite include of two or more materials, in which the material properties change smoothly from one interface to the other. During the past years, FG materials have a great practical importance because of their wide applications in many industrial and engineering fields. Recently, the application of FG materials has widely been developed in nano-structures such as nano-electromechanical systems, spacecraft heat shields, thin films in the form of shape memory alloys, plasma coatings for fusion reactors, and atomic force microscopes, jet fighter structures to obtain high sensitivity and desired function.

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Due to the presence of small-scale effects at the nanoscale, the classical continuum theories fail to accurately predict the mechanical behavior of nanostructures [1]. So, nonlocal continuum theories which contain additional material length scale parameter have been proposed to predict the accurate behavior of nano-structures. One widely promising size-dependent continuum theory is the nonlocal elasticity theory pioneered by Eringen [2].

The approximate analytical methods have their own restrictions. For example, Perturbation method depends on small parameter and choosing unsuitable small parameter can be lead to wrong solution [3]. Homotopy is an important part of topology [4] and it can convert any nonlinear problem to a finite linear problems and it doesn't depend on small parameter [5, 6]. Lia [7] based on homotopy proposed homotopy analysis method (HAM). The homotopy perturbation method was first proposed by Ji-Huan He [8] in 1999 for solving differential and integral equations. Homotopy perturbation method (HPM) is a combination of homotopy and classic perturbation techniques. The HPM has a significant advantage that it provides an analytical approximate solution to a wide range of nonlinear problems in applied sciences. HPM is a special case of HAM that due to its easier algorithm, it is used in this paper.

In practical, the governing differential equations of many vibrational systems are nonlinear such as satellites, helicopter blades, airplane wings, towers. They have no exact solution generally. Consequently, numerical or approximate analytical methods are used to investigate behavior of nonlinear systems. Numerical method like boundary element and finite element methods don't give parametric solutions. Hence, they have no application to study the qualitative and global response of the vibrational systems. Some approaches such as perturbation methods can eliminate shortages of numerical methods. There are some approaches to solve the governing equations of the nonlinear vibrations such as homotopy and perturbation methods and combination of them. We cite some of the papers that were used these methods.

Foda [9] and Ramezani et al. [10] used multiple scales to investigate the nonlinear vibration of the beam by considering shear deformation and rotary inertia effects for both sides simply supported (SS) and clamped-clamped, respectively. Nazemnezhad et al. [11] utilized the same method to investigate the nonlinear natural frequency of the FG nanobeam with considering small scale. The perturbation method was used to study the nonlinear vibrations of beams with different boundary conditions by Evensen [12]. Pirbodaghi et al. [13] studied nonlinear vibrational behavior of Euler Bernoulli beams subjected to axial loads using HAM. Akbarzade et al. [14] presented a new technique couples HPM with variational method for solving approximate analytical higher order solutions for strong nonlinear Duffing oscillators with cubic-quintic nonlinear restoring force. Bayat et al. [15] analyzed the high amplitude free vibrations of the tapered beams by using Max-Min Approach (MMA) and HPM. Ahmadian et al. [16] utilized homotopy and modified Lindstedt-Poincare methods to study the nonlinear free vibrations of the beams subjected to axial loads. Moeenfard et al. [17] used the same method to study the nonlinear natural frequencies of the pre-stretched microbeam considering the effects of