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XFEM for Fracture Analysis in 2D Anisotropic Elasticity

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Abstract. In this paper, a method is proposed for extracting fracture parameters in anisotropic thermoelasticity cracking via interaction integral method within the framework of extended finite element method (XFEM). The proposed method is applied to linear thermoelastic crack problems. The numerical results of the stress intensity factors (SIFs) are presented and compared with those reported in related references. The good agreement of the results obtained by the developed method with those obtained by other numerical solutions proves the applicability of the proposed approach and confirms its capability of efficiently extracting thermoelasticity fracture parameters in anisotropic materials.

AMS subject classifications: 65M60, 74A45

Key words: Anisotropic materials, thermoelasticity, crack, XFEM, stress intensity factors.

1 Introduction

Anisotropic materials, such as glass/epoxy, crystal, phenolic laminated sheet materials, etc., are widely used in many practical engineering structures due to their superior directional mechanical properties. During mechanical analysis and product design, it is generally necessary to consider anisotropic effects of the components produced from anisotropic materials. This has resulted in an upsurge of developing new fracture mechanical methods in anisotropic materials. The earliest paper about fracture problems in anisotropic materials was launched by Muskhelishvili [1]. Afterwards, G. C. Sih, P.

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C. Paris and G. R. Irwin investigated the crack problems in rectilinearly anisotropic bodies [2]. Studies of Z. Suo [3]solved some problems in singularities and cracks in dissimilar anisotropic medium. L. Nobile et al. [4] proposed an analytical method to study fracture problems in anisotropic/orthotropic medium. S. G. Lekhnitskii [5] researched on the elastic theory in anisotropic materials. S. Q. Zhang et al. [6] predicted the growth of Mode I crack in fiber reinforced composite. H. G. Jia, Y. F. Nie [7] proposed a method for simulating orthotropic thermoelasticity fracture. However, all above-mentioned works can only solve crack problems with simple geometry configuration and load. In view of this, numerical methods such as finite element method (FEM) [8, 9], boundary element method (BEM) [10–13], meshless method [14–16], and extended finite element method (XFEM) [17–20] have been developed to study more complicated fracture mechanical problems concerning mechanical and/or thermal load in anisotropic medium. Especially, in fracture of thermoelasticity in anisotropic medium, Y. C. Shiah et al. [10, 13] used BEM to analyze thermoelastic fracture in generally anisotropic materials. G. Hattori et al. [18] developed anisotropic crack-tip enrichment functions using Stroh's formalism within the framework of XFEM. S. S. Hosseini et al. [20] developed a new XFEM to analyze crack propagation in functionally graded materials. However, in these works, there is little attention paid to thermoelastic fracture analyzed by XFEM in fully anisotropic materials using anisotropic crack tip enrichment functions with Stroh's formalism.

In this paper, thermoelastic fracture analysis is performed via XFEM in fully anisotropic materials, and the SIFs around crack tips are obtained by domain formulation of interaction integral method combined with XFEM. On this basis, several numerical examples are given to validate the accuracy of results. To the best knowledge of the authors, it is the first time that applying XFEM with Stroh's formalism crack tip enrichment functions to fracture analysis of anisotropic composites under thermal loadings.

The rest of this paper is structured as follows. Fracture mechanics of anisotropic materials are given in Section 2. Section 3 states the governing equations of the problem and the discretization of the thermoelastic XFEM. Then, Section 4 gives the extraction of the SIFs from the XFEM by using domain formulation of interaction integrals with thermal effect. In Section 5, the proposed method is validated by several numerical examples and is compared with available solutions in literature. Finally, the conclusions from this research are summarized in Section 6.

2 Fracture mechanics of anisotropic materials

2.1 Constitutive relation of anisotropic materials

For anisotropic materials, the stress-strain relations [2] can be expressed as follows:

$$\varepsilon_i=\sum_{j=1}^{6}a_{ij}\sigma_j, \quad a_{ij}=a_{ji}, \quad (i=1,2,6), \tag{2.1}$$