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On the Factors Affecting the Accuracy and Robustness of Smoothed-Radial Point Interpolation Method

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Abstract. In order to overcome the possible singularity associated with the Point Interpolation Method (PIM), the Radial Point Interpolation Method (RPIM) was proposed by G. R. Liu. Radial basis functions (RBF) was used in RPIM as basis functions for interpolation. All these radial basis functions include shape parameters. The choice of these shape parameters has been and stays a problematic theme in RBF approximation and interpolation theory. The object of this study is to contribute to the analysis of how these shape parameters affect the accuracy of the radial PIM. The RPIM is studied based on the global Galerkin weak form performed using two integration technics: classical Gaussian integration and the strain smoothing integration scheme. The numerical performance of this method is tested on their behavior on curve fitting, and on three elastic mechanical problems with regular or irregular nodes distributions. A range of recommended shape parameters is obtained from the analysis of different error indexes and also the condition number of the matrix system. All resulting RPIM methods perform very well in term of numerical computation. The Smoothed Radial Point Interpolation Method (SRPIM) shows a higher accuracy, especially in a situation of distorted node scheme.

AMS subject classifications: 65N12, 97M50

Key words: Radial Basis Function, Radial Point Interpolation Methods, strain smoothing nodal integration, Galerkin weak form.

1 Introduction

One of most developed numerical techniques is the finite element method (FEM). In the FEM, a continuum solid is divided into a set of finite elements, the mesh of the solid,

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which are connected between them by nodes. The FEM proved to be effective and robust in several engineering fields because of its capacity in dealing with complex geometries. However, this method suffers from some limitations when severe element distortions take place under large deformation processes. In this context, the accuracy of results are lost [1]. To surmount this problem, meshless or meshfree methods are proposed where the problem domain is represented by a set of scattered nodes, without the need of any information about relationship between them. The development of some of the meshless methods goes back more than seventy years, with the appearance of collocation methods [3–5]. After that, the first meshless method known as the Smoothed Particle Hydrodynamics (SPH) [6], was originally used for the simulation of astrophysical phenomena by Lucy [6, 7]. From early 1990s, numerous methods have been proposed; for instance the diffuse element method (DEM) [8], the reproducing kernel particle method (RKPM) [9,10],the element free Galerkin (EFG) method [11], the point interpolation methods [12], the meshless local Petrov-Galerkin method (MLPG) [13]. All these methods use meshless shape functions to represent the field variables, since these shape functions are mathematically constructed by using only a set of nodes without requiring a mesh. The Moving least square (MLS) interpolation was one of the first shape functions used by Belytschko et al. [14] for the development of the element free Galerkin (EFG) method. Because some of its limitations, in particular, the complexity of the calculations of MLS shape functions and their partial derivatives, besides the difficulty of imposing boundary conditions [15]. Liu and Gu [16, 17] proposed a new family of meshless shape functions: "Point Interpolation Methods". Among these methods, the radial point interpolation method (RPIM), is preferred because the use of radial basis function avoids the problem of singularity such as conventional PIM [18, 55]. The shape functions resulting from RBF are stable and hence flexible for arbitrary and irregular nodal configurations. For numerical simulations of mechanics problems it is needed to combine shape function with formulation procedure based on strong or weak-forms derived directly from the physics. In meshless collocation methods (based on the strong-form), the PDE is usually discretized at nodes by some forms of collocation, therefore no background cells are required for numerical integration. There are various meshless based strong-form methods, e.g., the Finite Point Method (FPM) [49–51], the HP-Meshless Cloud Method [56] and the Radial Basis Function Collocation Methods (RBF-CM) [57–60], etc. In this paper, Galerkin weak form is used to construct discretized system equations. For the requirement of a weaker consistency on the approximate function, weak forms need an integral operation performed numerically by the use of two major techniques: the classical Gauss integration and the stabilized conforming nodal integration (SCNI) proposed by Beissel and Belytschko [19] and later by Chen et al. [20, 21]. The objective of the present work, is to study the RBF meshfree Galerkin Methods through their performances in term of: interpolations (RPIM shape function) and numerical integration techniques (classical Gauss integration and strain smoothing nodal integration). The paper is organized as follows. Firstly, an introduction on the used RBF shape functions and the integration schemes is given. A short outline of the algorithm used for the analysis is presented. After that, the