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A Simplified Lattice Boltzmann Method without Evolution of Distribution Function

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Abstract. In this paper, a simplified lattice Boltzmann method (SLBM) without evolution of the distribution function is developed for simulating incompressible viscous flows. This method is developed from the application of fractional step technique to the macroscopic Navier-Stokes (N-S) equations recovered from lattice Boltzmann equation by using Chapman-Enskog expansion analysis. In SLBM, the equilibrium distribution function is calculated from the macroscopic variables, while the non-equilibrium distribution function is simply evaluated from the difference of two equilibrium distribution functions. Therefore, SLBM tracks the evolution of the macroscopic variables rather than the distribution function. As a result, lower virtual memories are required and physical boundary conditions could be directly implemented. Through numerical test at high Reynolds number, the method shows very nice performance in numerical stability. An accuracy test for the 2D Taylor-Green flow shows that SLBM has the second-order of accuracy in space. More benchmark tests, including the Couette flow, the Poiseuille flow as well as the 2D lid-driven cavity flow, are conducted to further validate the present method; and the simulation results are in good agreement with available data in literatures.

AMS subject classifications: 76M28, 76P99

Key words: Chapman-Enskog expansion analysis, lattice Boltzmann equation, Navier-Stokes equations, memory cost, stability.

1 Introduction

Lattice Boltzmann Method (LBM) [1,2] is becoming a popular method in the field of Computational Fluid Dynamics (CFD). In the last few decades, LBM was being developed

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continuously [3–14], and has been widely applied in various kinds of fluid problems, including micro flows, thermal flows, multiphase flows and other problems [15–22].

Different from the conventional CFD methods based on the macroscopic Navier-Stokes (N-S) equations [23–27], LBM is a mesoscopic method rooted from the lattice Boltzmann equation (LBE) [28–31]. In LBM, the time marching is reflected in the evolution of the density distribution function. The macroscopic physical properties, such as the density and the velocity, are obtained from the conservation laws on a particular grid point. The evolution process of the density distribution function is realized by two steps: streaming and collision. Specifically, the streaming process is to distribute the effects from a local point to the surrounding points; and the collision process describes the combination effects on a local point. LBM is welcome by the CFD researchers due to several nice features. The first and also the most important feature is its kinetic nature. The simple streaming and collision processes in LBM are able to capture the complex nonlinear phenomenon in physics, and at the same time, avoid the manipulation of complex nonlinear terms or high order derivatives in macroscopic N-S equations. Secondly, LBM solves a set of algebraic equations; and no differential equations are involved, which makes the computation more straightforward and brief. In addition, being an explicit scheme, LBM facilitates the practical coding and parallelization, which makes it suitable to solve engineering problems. Apart from the above appealing characteristics, LBM also suffers from a number of drawbacks. Firstly, due to lattice uniformity, the standard LBM is only applicable on uniform mesh. To apply LBM on non-uniform mesh or for complex geometry, additional computational efforts are needed. The second drawback is that the standard LBM requires more virtual memories compared with the N-S solvers. It is needed to store the distribution functions along all lattice velocity directions at all grid points. Such storage requirement may be a heavy burden for large-scale problems, especially for 3D problems. Finally, it is also inconvenient for the standard LBM to implement the physical boundary conditions. The boundary conditions for the velocity and/or for the pressure need to be transformed into the conditions for the density distribution functions. Therefore, for problems with complex boundaries, it is not a simple task to implement appropriate boundary conditions.

To overcome the drawbacks of LBM, which is applied globally in the whole flow domain, a lattice Boltzmann flux solver (LBFS) [32] was recently developed, which only applies LBM locally. In LBFS, the Finite Volume Method (FVM) is applied to solve the macroscopic N-S equations, while the lattice Boltzmann method is implemented locally at each cell interface to calculate the fluxes. Based on the Chapman-Enskog (C-E) expansion analysis, the macroscopic flux can be evaluated from the LBE solutions. One of important contributions in LBFS is to approximate the non-equilibrium distribution function by the difference of two equilibrium distribution functions at two different locations and time levels. This way is much simpler than the conventional treatments with various expansion terms. Within the LBFS, only macroscopic flow variables are stored, and the physical boundary conditions can be easily implemented. At the same time, by introducing the LBM solver on the interface, the approximation of the high-order nonlin-