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Solution Reconstruction on Unstructured Tetrahedral Meshes Using P^1-Conservative Interpolation

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Abstract. This paper extends an algorithm of P^1-conservative interpolation on triangular meshes to tetrahedral meshes and thus constructs an approach of solution reconstruction for three-dimensional problems. The conservation property is achieved by local mesh intersection and the mass of a tetrahedron of the current mesh is calculated by the integral on its intersection with the background mesh. For each current tetrahedron, the overlapped background tetrahedrons are detected efficiently. A mesh intersection algorithm is proposed to construct the intersection of a current tetrahedron with the overlapped background tetrahedron and mesh the intersection region by tetrahedrons. A localization algorithm is employed to search the host units in background mesh for each vertex of the current mesh. In order to enforce the maximum principle and avoid the loss of monotonicity, correction of nodal interpolated solution on tetrahedral meshes is given. The performance of the present solution reconstruction method is verified by numerical experiments on several analytic functions and the solution of the flow around a sphere.

AMS subject classifications: 41A05, 65D05

Key words: Solution reconstruction, solution transfer, conservative interpolation, tetrahedral mesh, solution interpolation, mesh intersection.

1 Introduction

Solution reconstruction via interpolation from one mesh to another is absolutely vital in many numerical simulations, such as multi-physics simulations [1], fluid-structure interaction problems, numerical weather prediction and climate simulation [2–4] and mesh adaptation for time-dependent problems [5]. As repeated interpolation is needed for

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these problems, the accuracy and conservation of the interpolation are very important. Maintaining the conservation of the interpolation can avoid the excessive accumulation of errors [5, 6]. Therefore, some interpolation methods preserving the conservation are developed.

The remapping process of arbitrary Lagrangian-Eulerian (ALE) method is one of the important applications of conservative interpolation. In Lagrangian step, points and cells move together with the flow until certain cells become extremely distorted, then a rezoning procedure is performed. Therefore, a remapping procedure to transfer data from one mesh to another is needed. The related literatures on remapping can be found in [7–9]. Meshes used in ALE method are usually quadrilateral or hexahedral meshes with a fixed topology. A conservative remapping scheme for grids in spherical coordinates is proposed by Jones [10]. Some of these methods have been extended to the application for two unrelated meshes [11, 12]. Integral (or global) remapping permits that the current mesh and the background mesh are unrelated. Therefore, it has complete flexibility for the two meshes at the cost of calculating the integral volume of the intersection of background and current cells. A conservative interpolation method that converts the volume integral to a surface integral by using the divergence theorem is proposed by Dukowicz [13]. This method can simplify the problem of volume intersection computing, but it suffers from excessive diffusion due to its first order nature. Some work has been done to improve the accuracy by Dukowicz and Kodis in [7]. Geuzaine et al. proposed a Galerkin projection method that can be used to mesh-to-mesh interpolation [14]. A common-refinement based data transfer method applicable to surface meshes is presented by Jiao and Heath [1]. By using a similar concept with common-refinement, a conservative interpolation method based on Galerkin projection is proposed by Farrell et al. in [15, 16]. This method can be used for two unrelated volume meshes, but it involves complex matrix operations and the construction of the so-called supermesh.

In [5], Alauzet and Mehrenberger proposed an effective two-dimensional P^1-conservative interpolation algorithm. The mass conservation property of the interpolation operator is achieved by local triangle-triangle intersection. For a given solution on the background mesh, the mass and gradient of the interpolated solution on each triangle of the current mesh is first computed from those on all triangles of background mesh it overlaps. Then, the solution on each node of the current triangle can be reconstructed by using the mass and gradient transferred from the background mesh. In order to avoid the loss of monotonicity, the maximum principle is enforced by a specific correction of the interpolated solution. This algorithm is P^1 exact (order 2) and permits that the current mesh and the background mesh are unrelated while the interpolation algorithms used in the ALE methods are usually for quadrilateral and hexahedral meshes with a fixed topology (unchanged connectivity) [7–9] or only of the first order accuracy [13]. Compared to the algorithm of Farrell et al. [15, 16] in which resolution of a linear system and construction of the super-mesh are required, it is a matrix-free approach based on local mesh intersections and appropriate local reconstructions. The locality is primordial for efficiency and robustness. The conservation property of this algorithm has been checked in [5]. Howev-