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Modeling and Computation of CO_2 Allowance Derivatives Under Jump-Diffusion Processes

Shuhua Zhang[1,2,*] and Jing Wang[3,1]

[1] *Research Center for Mathematics and Economics, Tianjin University of Finance and Economics, Tianjin 300222, China*
[2] *Institute of Policy and Management, Chinese Academy of Sciences, Beijing 100190, China*
[3] *Department of Mathematics and Physics, Bengbu College, Bengbu, 233030, China*

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Abstract. In this paper, we study carbon emission trading whose market is gaining in popularity as a policy instrument for global climate change. The mathematical model is presented for pricing options on CO_2 emission allowance futures with jump diffusion processes, and a so-called fitted finite volume method is proposed to solve the pricing model for the spatial discretization, in which the Crank-Nicolson is employed for time stepping. In addition, the stability and the convergence of the fully discrete scheme are given, and some numerical results, which are compared with the closed form solution and the Monte Carlo simulation solution, are provided to demonstrate the rates of convergence and the robustness of the numerical method.

AMS subject classifications: 65M12, 65M60, 91B28

Key words: CO_2 emission allowance, option pricing, jump diffusion, fitted finite volume method, partial integro-differential equation, fast Fourier transform.

1 Introduction

Global climate warming and its dangerous ecological, social, and economic consequences have gained increasing public attention in recent years. The increasing concentration of greenhouse gases (GHGs) in the atmosphere over the past century is to some extent related to human activities, among which, first of all, the emitted CO_2 characterized by an essential increase in human fossil fuel consumption is. Now carbon emission allowance trading has been accepted as a way to reduce emissions at the lowest abatement cost by adding economic cost to carbon emissions, which is an environmental policy device.

*Corresponding author.
Email: shuhua55@126.com (S. H. Zhang), crysralstella@126.com (J. Wang)

Up to now, emission trading schemes have been designed by introducing appropriate market mechanisms. For example, the European Union emission trading scheme (EU ETS), a cap and trade system, is the most prominent example. Within this scheme, each member state receives a fixed amount of EUA (European Union Allowance), which is called the cap. The European Union started with a pilot period, 2005-2007, followed by the Kyoto commitment period 2008-2012. The third period will last from 2013 to 2020.

Also, the same efforts are being made in China. In fact, the emissions trading platforms have opened in China in the cities/provinces undertaking pilot stage GHG mitigation, which are exclusive and are not yet linked. They will be linked within the nationwide emissions trading system after 2015 or 2016. China's trading programs are now becoming the second largest emissions trading scheme following the EU ETS. Before the emissions trading platforms opened, China had already experienced with the carbon market through the CERs (Certified Emissions Reduced) produced by the Clean Development Mechanism (CDM) projects under the Kyoto Protocol. In addition, in October of 2011, the National Development Reform Commission (NDRC) designated seven provinces and cities, Guangdong, Hubei, Beijing, Shanghai, Tianjin, Chongqing, and Shenzhen, as regional mandatory ETS pilots (see Fig. 1). Most of the pilot regions should submit their ETS implementation plans to the NDRC. A national ETS is also under consideration.

Similar to all other financial or commodity markets, both regulated companies and investors are subject to carbon trading risks and need suitable risk management tools. As a result, some carbon derivatives have been developed. At the moment futures and forwards on carbon emission allowances are fairly standard and are traded both over the counter (OTC) and on exchanges. Moreover, in October of 2006, European-style call and put options on EUA futures started trading in the European Climate Exchange (ECX). In

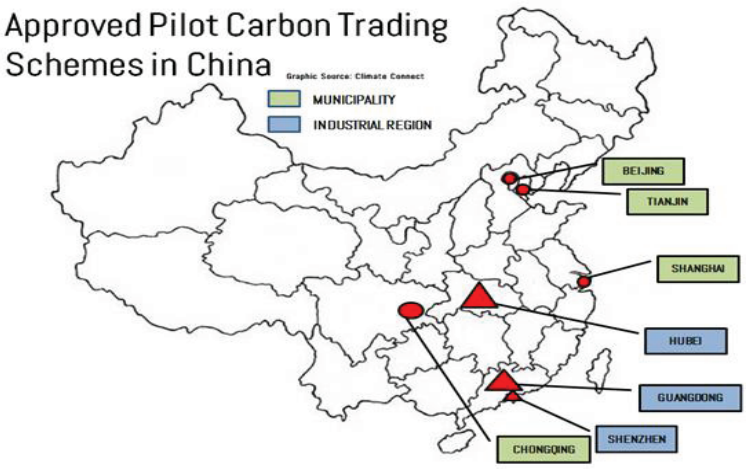

Figure 1: The mandatory emissions trading cities/regions in China (The source is from P. R. Shukla's report on "Emissions Cap and Trade: Implemen1tation Issues, Lessons and Recommendations").