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High-Order Mesh Generation for Discontinuous Galerkin Methods Based on Elastic Deformation

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Abstract. In this paper, a high-order curved mesh generation method for Discontinuous Galerkin methods is introduced. First, a regular mesh is generated. Second, the solid surface is re-constructed using cubic polynomial. Third, the elastic governing equations are solved using high-order finite element method to provide a fully or partly curved grid. Numerical tests indicate that the intersection between element boundaries can be avoided by carefully defining the elasticity modulus.

AMS subject classifications: 35L67, 65M60

Key words: Curved mesh generation, discontinuous Galerkin methods, elastic deformation, Lagrangian finite element.

1 Introduction

The necessity of using curved solid boundary when using high-order DG (Discontinuous Galerkin) methods to solve the steady-state Euler equations was first reported by Bassi in 1997 [1]. Numerical tests indicated that high-order DG is very sensitive to the representation of solid boundary surfaces and the curved solid boudary which can accurately approximate the real physical geometry is required to enssure the convergence when $p \geq 2$.

The way to implement curved solid boundary is not unique. In 2D case, The most straightforward method is to curve the element boudaries on the solid suface only and keep the others straight [1], which generates minimum curved elements and saves CPU time since the numerical integration over curved element or curved element boundary is relatively expensive compared to straight case. In 2005, Krivodonova and Berger introduced several ways to implement the solid boundary condition for high-order DG methods in 2D case [6]. A boundary representation method based on the Bezier curve

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is proposed for 2D high-order Euler solvers in [4]. Quadratic polynomials are used to describe the geometry when using 2D DG to solve aeroacoutic problems in [10]. A 3d high-order boundary construction method is developed in [7]. Persson and Peraire introduced a curved unstructured mesh generation method based on nonlinear elesticity analogy which generates non-intersecting elements if the initial mesh is fine enough to resolve the solid deformation [8].

In 3D case, all of the interfaces of the tetrahedron elements on the solid boundary must be treated as curved faces besides the ones on the solid boundary, which means at least the interior elements neighboring to the boundary elements are also curved [7]. However, in both 2D and 3D case, intersection could happen when the boundary faces are curved too much compared to the element size [8], which requires the original grid points are carefully placed.

In this paper, a new curved mesh generation method is developed. The final curved mesh is obtained by solving a linear elastic governing equations [9] with high-order finite element method on a given regular mesh. Non-intersecting elements can be guaranteed since the computational domain is treated as an elastic solid. More importantly, this method can further improve the quality of the entire mesh structure by appropriately defining the elastic modulus if needed.

2 Governing equations for elastic deformation

The governing equations for elastic deformation used in this paper is as follows:

$$L_u(u,v,e)=\frac{\partial}{\partial x}\Big(e\frac{\partial U}{\partial x}\Big)+\alpha\frac{\partial}{\partial x}\Big(e\frac{\partial V}{\partial y}\Big)+\beta\frac{\partial}{\partial y}\Big(e\frac{\partial U}{\partial y}+e\frac{\partial V}{\partial x}\Big)=0, \tag{2.1a}$$

$$L_v(u,v,e)=\beta\frac{\partial}{\partial x}\Big(e\frac{\partial U}{\partial y}+e\frac{\partial V}{\partial x}\Big)+\frac{\partial}{\partial y}\Big(e\frac{\partial V}{\partial y}\Big)+\alpha\frac{\partial}{\partial y}\Big(e\frac{\partial U}{\partial x}\Big)=0, \tag{2.1b}$$

where $\alpha=\nu/(1-\nu)$, $\beta=(1-\alpha)/2$ and e and ν are the modulus of elasticity and the Poisson ratio respectively. The unknown U and V are the displacements which can be written as:

$$U(x,y)=X(x,y)-x, \tag{2.2a}$$

$$V(x,y)=Y(x,y)-y, \tag{2.2b}$$

where $X(x,y)$ and $Y(x,y)$ are the new co-ordinates after the displacement. Eq. (2.1) can be re-written as:

$$L_u(X,Y,e)=b_u, \tag{2.3a}$$

$$L_v(X,Y,e)=b_v, \tag{2.3b}$$

where

$$b_u=L_u(x,y,e)=0, \tag{2.4a}$$

$$b_v=L_v(x,y,e)=0, \tag{2.4b}$$