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Brittle and Ductile Character of Amorphous Solids

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Abstract. Common silicate glasses are among the most brittle of the materials. However, on warming beyond the glass transition temperature T_g glass transforms into one of the most plastic known materials. Bulk metallic glasses exhibit similar phenomenology, indicating that it rests on the disordered structure instead on the nature of the chemical bonds. The micromechanics of a solid with bulk amorphous structure is examined in order to determine the most basic conditions the system must satisfy to be able of plastic flow. The equations for the macroscopic flow, consistent with the constrictions imposed at the atomic scale, prove that a randomly structured bulk material must be either a brittle solid or a liquid, but not a ductile solid. The theory permits to identify a single parameter determining the difference between the brittle solid and the liquid. However, the system is able of perfect ductility if the plastic flow proceeds in two dimensional plane layers that concentrate the strain. Insight is gained on the nature of the glass transition, and the phase occurring between glass transition and melting.

AMS subject classifications: 74A45, 74A60, 74A20

Key words: Glasses, bulk metallic glass, shear bands, ductility, micromechanical modelling.

1 Introduction

The plastic deformation of amorphous metallic alloys exhibiting ductile behaviour proceeds primarily through the formation of thin layers or shear bands, of typical thicknesses going from tens of nanometers to some microns, which concentrate the strain [1,2]. Fracture generally occurs by decohesion of one of these shear bands crossing the sample at an angle of 45° with respect to the uniaxial loading direction. In contrast to most crystalline metals, no strain hardening is registered, and instances of perfect plasticity have been reported [1]. However, most amorphous metals developed so far do not show this

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rather special mechanism of deformation and undergo brittle fracture on just reaching the elastic limit [3].

The observed properties of bulk metallic glasses shed light on a long lasting enigma concerning common silicate glasses. Common glasses are among the most brittle of the materials. A sign representing a glass cup in a freight package universally warns about its fragile content. However, on warming beyond the glass transition temperature T_g, what resembles a second order phase transition takes place and glass transforms into a highly plastic material, allowing for the glassblowing fabrication of light complex shaped vessels or extremely fine capillary tubes by stretching hot enough pipes. The point is that most bulk metallic glasses exhibit similar behaviour, and, together with their essentially brittle character, evidence of a glass transition has been observed in many of them [2,4–6]. As the cohesive forces of amorphous metals can be still associated to delocalized, or at least poorly localized, electrons, this shows that the extreme brittleness and plasticity of common silicate glasses cannot be attributed to the covalent nature of their atomic bonds. Instead, brittleness and glass transition seem to have their physical origin in the random character of the microscopic structure of the material. However, the random structure cannot explain by itself the phenomenology of glasses because liquids are amorphous as well.

This article intends to identify the missing ingredient in order to explain why a material structured as a three dimensional continuous random network must be brittle, and why the formation of layers may yield ductile behaviour. For this aim the traditional approach to the problem is changed radically and it is assumed first, as a working hypothesis, that the material is able of plastic deformation to determine the conditions that should be met to make this effective. Also, an insight in the nature of the thermodynamic state occurring in between the glass transition and melting is gained from considering bound states of the randomly structured bulk material at high temperatures.

The present theoretical approach to the study of amorphous materials has not been reported before, but the general idea was advanced in a previous paper by the first author [7]. Following the pioneering investigations of [8, 9] and [10], most investigations on the fracture of solids ascribe brittle behavior to the presence of preexisting cracks that can propagate across the material conserving their atomically sharp leading edges. In ductile solids the tip of the crack should become blunt and broad because of the plastic deformation ahead of it, increasing effort to make it progress [11–18]. The problem of stress induced crack propagation has been studied by theory and computer simulations [19–21] in order to ascertain the origin of brittle or ductile fracture and the dynamics of the fracture process. The complex evolution of crack growth has been accurately measured [22–24], confirming atomic scale model predictions [25, 26] that the dynamics of a crack tip is highly unstable, and steady motion in a given direction is in most situations impossible. We propose here an alternate point of view, concerning the initiation of brittle fracture.

The next sections show that the question of **why** a solid breaks has a much more basic and simpler answer than **how** fracture proceeds. Resorting to a very general model for