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Time-Domain Numerical Solutions of Maxwell Interface Problems with Discontinuous Electromagnetic Waves

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Abstract. This paper is devoted to time domain numerical solutions of two-dimensional (2D) material interface problems governed by the transverse magnetic (TM) and transverse electric (TE) Maxwell's equations with discontinuous electromagnetic solutions. Due to the discontinuity in wave solutions across the interface, the usual numerical methods will converge slowly or even fail to converge. This calls for the development of advanced interface treatments for popular Maxwell solvers. We will investigate such interface treatments by considering two typical Maxwell solvers – one based on collocation formulation and another based on Galerkin formulation. To restore the accuracy reduction of the collocation finite-difference time-domain (FDTD) algorithm near an interface, the physical jump conditions relating discontinuous wave solutions on both sides of the interface must be rigorously enforced. For this purpose, a novel matched interface and boundary (MIB) scheme is proposed in this work, in which new jump conditions are derived so that the discontinuous and staggered features of electric and magnetic field components can be accommodated. The resulting MIB time-domain (MIBTD) scheme satisfies the jump conditions locally and suppresses the staircase approximation errors completely over the Yee lattices. In the discontinuous Galerkin time-domain (DGTD) algorithm – a popular Galerkin Maxwell solver, a proper numerical flux can be designed to accurately capture the jumps in the electromagnetic waves across the interface and automatically preserves the discontinuity in the explicit time integration. The DGTD solution to Maxwell interface problems is explored in this work, by considering a nodal based high order discontinuous Galerkin method. In benchmark TM and TE tests with analytical solutions, both MIBTD and DGTD schemes achieve the second order of accuracy in solving circular interfaces. In comparison, the numerical convergence of the MIBTD method is slightly more uniform, while the DGTD method is more flexible and robust.

AMS subject classifications: 65M06, 65M60, 78M10, 78M20

Key words: Maxwell's equations, finite-difference time-domain (FDTD), discontinuous Galerkin time-domain (DGTD), transverse magnetic (TM) and transverse electric (TE) systems, high order interface treatments, matched interface and boundary (MIB).

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1 Introduction

When the permittivity and permeability coefficients are discontinuous across a material interface separating two dielectric media, the electric and magnetic field components could be discontinuous. Without a proper interface treatment, the time domain numerical solution of Maxwell's equations that govern the propagation and scattering of electromagnetic waves in nonhomogeneous media converges slowly or even fails to converge [12]. Because the material interfaces are omnipresent in optical devices, microwave circuits, antennas, aircraft radar signature, nano/micro electric devices and telecommunication chips, the development of innovative computational methods for dealing with electromagnetic interface problems with discontinuous solutions has received much attention in recent years. Various different interface strategies have been developed in the computational electromagnetics (CEM) to cope with the problems caused by the loss of solution regularity and the complex geometry of the material interface [12].

It is well known that the finite-difference time-domain (FDTD) method based on the Yee lattice has been a main workhorse of the CEM in the time domain [34], owing to its simplicity, free of dissipative error and having very low cost per grid node. However, the standard FDTD algorithm suffers from a large accuracy reduction at a dielectric interface, due to not only a staircase representation for complex geometry, but also the lack of the enforcement of solution jumps in the numerical discretization. To overcome this difficulty, several embedded FDTD methods [3,8,12,36,37,40,41,46] have been successfully developed to restore the accuracy near a dielectric interface. Computationally, some sophisticated interface treatments are conducted near the interface or on a one-dimensional smaller set of nodes to rigorously impose the jump conditions in the FDTD discretizations, while away from the interface, the standard FDTD scheme can be employed. While maintaining the simplicity and computational efficiency of the Yee scheme [34], the embedded FDTD methods can fully restore second order accuracy, even in case of curved boundaries and interfaces, by using a simple Cartesian grid. Nevertheless, Maxwell interface problems with continuous wave solutions are normally considered in these embedded FDTD methods. The discontinuous Maxwell interface problem to be attacked in this work is less well studied in the FDTD literature.

Recently, we have developed a family of finite difference algorithms based on the matched interface and boundary (MIB) method [42,47] for solving regular dielectric interface problems [42,43] and dispersive interface problems [29,30,44]. The MIB interface treatments are quite flexible, through the introduction of fictitious values and an iterative use of zeroth and first order jump conditions [42,47]. The MIB modeling is systematically carried out and can be made to arbitrarily high order in principle in the presence of straight material interfaces. Orders up to 12 have been achieved numerically for solving Maxwell's equations [42]. When dispersive materials are considered, the material constitutive coefficients are functions of frequency so that the jumps in electromagnetic fields become time variant across dispersive interfaces. The MIB time-domain (MIBTD) method [29,30,44] is the only known second order accurate FDTD algorithm for solving