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Stability Analysis and Order Improvement for Time Domain Differential Quadrature Method

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Abstract. The differential quadrature method has been widely used in scientific and engineering computation. However, for the basic characteristics of time domain differential quadrature method, such as numerical stability and calculation accuracy or order, it is still lack of systematic analysis conclusions. In this paper, according to the principle of differential quadrature method, it has been derived and proved that the weighting coefficients matrix of differential quadrature method meets the important **V**-transformation feature. Through the equivalence of the differential quadrature method and the implicit Runge-Kutta method, it has been proved that the differential quadrature method is A-stable and s-stage s-order method. On this basis, in order to further improve the accuracy of the time domain differential quadrature method, a class of improved differential quadrature method of s-stage $2s$-order has been proposed by using undetermined coefficients method and Padé approximations. The numerical results show that the proposed differential quadrature method is more precise than the traditional differential quadrature method.

AMS subject classifications: 37M05, 65L05, 65L06, 65L20

Key words: Differential quadrature method, numerical stability, order, **V**-transformation, Runge-Kutta method, Padé approximations.

1 Introduction

The differential quadrature method (DQM) was first proposed by Bellman and his associates in the early 1970s [1,2], which is used for solving ordinary and partial differential equations. As an analogous extension of the quadrature for integrals, it can be essentially expressed as the values of the derivatives at each grid point as weighted linear sums approximately of the function values at all grid points within the domain under consideration.

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The differential quadrature method is conceptually simple and the implementation is straightforward. It has been recognized that the differential quadrature method has the capability of producing highly accurate solutions with minimal computational effort [3, 4] when the method is applied to problems with globally smooth solutions. So far, the differential quadrature method has been widely applied to boundary-value problems in many areas of engineering and science, such as transport process [5], structural mechanics [6–8], calculation of transmission line transient response [9, 10], etc. [11] made the first attempt to apply the differential quadrature method for time domain discretization. Subsequently, the differential quadrature method has been extended to solve initial value problems in the time domain, such as, time-dependent diffusion problems [12], transient elastodynamic problems [13] and dynamic problems [14–16]. A comprehensive review of the chronological development of the differential quadrature method can be found in [4, 11]. Though the differential quadrature method has been successfully applied in so many fields, for the basic characteristics of the method, such as numerical stability and calculation accuracy or order, not much work about it has been done in this area for the differential quadrature method. According to Fung [17], using Lagrange interpolation functions as test functions, the time domain differential quadrature has been shown to be equivalent to the recast implicit Runge-Kutta method [18–20], besides, some low-order algorithms were discussed in detail. However, the method used by Fung is not the traditional sense of differential quadrature method, but involved post-processing (i.e., numerical solution at the end of grid points adopts polynomial extrapolation).

In this paper, using general polynomial as test functions [21], the weighting coefficients matrix of differential quadrature method is proved to satisfy **V**-transformation [19, 22]. The equivalent implicit Runge-Kutta method is constructed through the differential quadrature method. Hence, making use of Butcher fundamental order theorem and the method of linear stability analysis [18–20], the basic characteristics of differential quadrature method can be systematically analysed. Unfortunately, the differential quadrature method is only the method of s-stage s-order and A-stable. Consequently, the differential quadrature method can't yield higher accurate solutions to the boundary-value problems with fewer computational efforts. Based on above deduction, the method of undetermined coefficients is used to make the stability function of the equivalent Runge-Kutta method become the diagonal Padé approximations to the exponential function [19, 20]. Therefore, a class of improved differential quadrature method of s-stage $2s$-order has been derived.

The manuscript is arranged as follows. In Section 2, the weighting coefficients matrix of traditional differential quadrature method using general polynomial as test functions is briefly discussed. In Section 3, the equivalent relationship between differential quadrature method and Runge-Kutta method is deduced. In Section 4, the stability and order characteristics of differential quadrature method are studied. A class of improved differential quadrature method of s-stage $2s$-order and A-stable is proposed in Section 5. In Section 6, the transient response of a double-degree-of-freedom system is computed, which is given to verify the computational accuracy with the defined three grid points.