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An Efficient Semi-Analytical Method to Compute Displacements and Stresses in an Elastic Half-Space with a Hemispherical Pit

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Abstract. This paper presents an efficient method to calculate the displacement and stress fields in an isotropic elastic half-space having a hemispherical pit and being subject to gravity. The method is semi-analytical and takes advantage of the axisymmetry of the problem. The Boussinesq potentials are used to obtain an analytical solution in series form, which satisfies the equilibrium equations of elastostatics, traction-free boundary conditions on the infinite plane surface and decaying conditions at infinity. The boundary conditions on the free surface of the pit are then imposed numerically, by minimising a quadratic functional of surface elastic energy. The minimisation yields a symmetric and positive definite linear system of equations for the coefficients of the series, whose particular block structure allows its solution in an efficient and robust way. The convergence of the series is verified and the obtained semi-analytical solution is then evaluated, providing numerical results. The method is validated by comparing the semi-analytical solution with the numerical results obtained using a commercial finite element software.

AMS subject classifications: 35Q74, 35C10, 74G15

Key words: Elastic half-space, semi-infinite domain, axisymmetric domain, hemispherical pit, semi-analytical solution.

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1 Introduction

Boundary-value problems of linear elastostatics in unbounded domains are of current interest in various disciplines related to earth sciences, such as geophysics, soil and rock mechanics, structural and foundation engineering, and mining engineering, among others. To determine stresses and displacements around certain structures of interest such as tunnels, cracks, or underground and open-pit excavations, it is necessary, in order to produce accurate results, to take into account the surrounding medium, which for modelling purposes is assumed to be an unbounded elastic domain. In particular, the main motivation of the present investigation comes from the need for calculating the stresses and displacements induced by gravity in a mining excavation, in order to determine the areas in risk of collapse due to high-stress concentration. For this, the surrounding rock mass needs to be taken into account.

To define a methodology for solving this type of problems, the unboundedness of the domain represents a major difficulty. Numerical methods appear to be a good alternative to solve boundary-value problems in general, however, most of them require a bounded computational domain. This problem is often solved by truncating artificially the unbounded domain, reducing the original problem to a bounded domain. Nevertheless, artificial boundary conditions have to be imposed on its boundary in order to obtain correct results. The choice of such boundary conditions is not clear a priori, constituting this issue in itself a whole area of research. Another alternative is to calculate an analytical solution to the unbounded boundary-value problem. Even though it is well-known that analytical solutions have the disadvantage of being possible only for certain simple geometries, they are useful since they can be used to deduce artificial boundary conditions for a truncated domain, which is then discretized numerically. Some authors who have followed this approach in linear elastostatics are Han and Wu [13, 14], Givoli and Keller [9], and Givoli and Vigdergauz [10].

Most of the analytical solutions for unbounded elastic domains available in the literature are for exterior domains with simple geometries. In particular, the solutions provided in [9, 13, 14] are for the exterior of a circle. Even though these analytical solutions may be used in applications related to geosciences, a more precise approximation of the unbounded medium that surrounds an excavation (or another similar structure) is achieved with a half-plane or a half-space, with the excavation represented as a local perturbation. This type of domain is called semi-infinite and has the difficulty of being bounded by an infinite plane surface, which is assumed to be traction-free in most cases. Nevertheless, analytical solutions are still possible for some simple geometries. In the two-dimensional case, certain complex variable methods have proven to be useful for that purpose. Givoli and Vigdergauz [10] employed Kolosoff-Muskhelishvili potentials to solve the elastostatic problem at the exterior of a semicircle in a half-plane and used the obtained solution to derive artificial boundary conditions for geophysical applications. A similar technique was applied by Verruijt [18, 19] to solve the problem of an elastic half-plane with an embedded circular cavity.