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A Comparison and Unification of Ellipsoidal Statistical and Shakhov BGK Models

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Abstract. The Ellipsoidal Statistical model (ES-model) and the Shakhov model (S-model) were constructed to correct the Prandtl number of the original BGK model through the modification of stress and heat flux. With the introduction of a new parameter to combine the ES-model and S-model, a generalized kinetic model can be developed. This new model can give the correct Navier-Stokes equations in the continuum flow regime. Through the adjustment of the new parameter, it provides abundant dynamic effect beyond the ES-model and S-model. Changing the free parameter, the physical performance of the new model has been tested numerically. The unified gas kinetic scheme (UGKS) is employed for the study of the new model. In transition flow regime, many physical problems, i.e., the shock structure and micro-flows, have been studied using the generalized model. With a careful choice of the free parameter, good results can be achieved for most test cases. Due to the property of the Boltzmann collision integral, the new parameter in the generalized kinetic model cannot be fully determined. It depends on the specific problem. Generally speaking, the S-model predicts more accurate numerical solutions in most test cases presented in this paper than the ES-model, while ES-model performs better in the cases where the flow is mostly driven by temperature gradient, such as a channel flow with large boundary temperature variation at high Knudsen number.

AMS subject classifications: 65M10, 78A48

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1 Introduction

The monatomic rarefied gas behavior can be described by the Boltzmann equation. However, the collision term of the Boltzmann equation is a multiple integral term which is

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very complicated for analysis and numerical computation. Bhatnagar et al. [1] simplified the collision term in the Boltzmann equation and proposed BGK model in which the Boltzmann collision term is replaced by a relaxation term. This relaxation term mimics the main relaxation process from nonequilibrium state towards to a local equilibrium one. The local equilibrium state known as Maxwell distribution function is determined by the local conservative flow variables, namely, the mass, momentum and energy. Due to its simplicity, the BGK model becomes an important kinetic model for analysis and numerical simulation of nonequilibrium flows. However, the Chapman-Enskog expansion of the BGK model derives the Navier-Stokes equations with a unit Prandtl number, which is different from the physical reality in the continuum flow regime. For a monatomic gas, the accepted Prandtl number is about 2/3 in a wide range of flow conditions.

In order to fix the Prandtl number, many kinetic models have been proposed in the past decades. The main approach is to modify the relaxation term. For example, the Ellipsoidal Statistical BGK model [2] employs a Gaussian distribution as the relaxation equilibrium state instead of the Maxwell distribution. This model is not very popular until Andries [3] proved the entropy condition of the ES-model. In the ES-model, besides the conservative flow variables, the local stress tensor also involves in the relaxation term. By changing the free parameter in the ES-model, it can present an arbitrary Prandtl number. Moreover, the nonnegative property of the Gaussian distribution is a favorable physical property.

Another very popular kinetic model is the Shakhov model [4]. Unlike the ES-model, it adjusts the heat flux in the relaxation term. With the implementation of Hermite polynomial, the low order moments of relaxation term in S-model are identical to the original BGK one, namely, the conservative variables are maintained and the stress tensor keeps isotropic one as that from the BGK model. Only the third order moments of relaxation term change. In other words, the S-model modifies the BGK model by adjusting heat flux to present the correct Prandtl number. But, it allows negative value of distribution function and its H-theorem was only proved in near equilibrium condition [4].

In 1990, Liu [5] proposed a new kinetic model by considering the gain term and lost term of the Boltzmann equation separately, where the Chapman-Enskog distribution is directly used to evaluate the relaxation term. Therefore, the space derivatives are involved in the collision term. Liu model changes both the heat flux and stress tensor of the relaxation process and provides a correct Prandtl number in the continuum flow regime. Due to its relatively complicated formulation, this model has not been widely used.

Although all above models provide correct Prandtl number in the continuum flow regime, their properties are very different in the transition regime [6–10]. Garzo [7] reported a singular behavior of Liu model and attributed it to the negative distribution function. Graur [8] studied the heat transfer problem and found that the ES-model provides better results than the S-model through the comparison with the results from the Boltzmann equation. The ES-model keeps the distribution function positive, while the S-model and Liu model always allow un-physical negative distribution function. It seems