East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.260316.061016a

Vol. **7**, No. 2, pp. 227-247
May 2017

Finite Volume Method for Pricing European and American Options under Jump-Diffusion Models

Xiao-Ting Gan[1,2], Jun-Feng Yin[1,*] and Yun-Xiang Guo[1]

[1] *School of Mathematical Science, Tongji University, Shanghai 200092, PR China.*
[2] *School of Mathematics and Statistics, Chuxiong Normal University, Chuxiong 675000, Yunnan Province, PR China.*

Received 26 March 2016; Accepted (in revised version) 6 October 2016.

Abstract. A class of finite volume methods is developed for pricing either European or American options under jump-diffusion models based on a linear finite element space. An easy to implement linear interpolation technique is derived to evaluate the integral term involved, and numerical analyses show that the full discrete system matrices are M-matrices. For European option pricing, the resulting dense linear systems are solved by the generalised minimal residual (GMRES) method; while for American options the resulting linear complementarity problems (LCP) are solved using the modulus-based successive overrelaxation (MSOR) method, where the H_+-matrix property of the system matrix guarantees convergence. Numerical results are presented to demonstrate the accuracy, efficiency and robustness of these methods.

AMS subject classifications: 65M08, 65M12, 65M60, 65N40

Key words: Finite volume method, option pricing, jump-diffusion models, linear complementarity problems, GMRES method, modulus-based successive overrelaxation method.

1. Introduction

The standard Black-Scholes equation has been successfully and widely used in finance, since the underlying model and related assumptions are simple and elegant [5]. However, empirical findings have shown that the standard Black-Scholes assumption of lognormal stock diffusion with constant volatility is not consistent with market prices. This is often referred to as the volatility skew or smile, and occurs in all major stock markets. In order to address this phenomenon, researchers have made various extensions to the Black-Scholes model. The jump-diffusion model first introduced by Merton allows for jump risks on option prices, and can account for large price changes due to rare events. Unlike in the original Black-Scholes model, the price is then intrinsically no longer a continuous function of time,

*Corresponding author. *Email addresses:* `9xtgan@tongji.edu.cn` (X.-T. Gan), `yinjf@tongji.edu.cn` (J.-F. Yin), `103646@tongji.edu.cn` (Y.-X. Guo)

but implied volatility curves yielded by jump-diffusion models are closer to the volatility smiles observed on markets.

It is challenging to provide numerical methods to solve partial integral differential equations (PIDE) for pricing options under jump-diffusion models, since a non-local integral term is involved [1, 2, 10, 11, 15]. For either European or American options under Kou's jump-diffusion models, Toivanen [13] recently developed an easy-to-implement recursion formula for the integral terms with optimal computational cost. A generalised iterative method for the linear complementarity problem (LCP) with a banded matrix instead of the full matrix has been provided by Salmi *et al.* [12]. Toivanen [14] has also proposed a high-order front-tracking finite difference method for pricing American options under jump-diffusion models, giving an easy way to implement fourth-order accurate discretisations as well. A fitted finite volume method was presented for the numerical solution of European and American jump-diffusion models by Zhang *et al.* [16, 17], essentially based on a finite volume formulation with a fitted local approximation to the solution. The finite volume approach usually possesses a special feature of local conservation of the numerical fluxes, and has been gaining popularity [9].

Numerical methods for pricing European and American options under Merton's jump-diffusion models are further studied here. We apply the finite volume scheme in space, and the backward Euler and Crank-Nicolson schemes in time. For European options, the resulting dense linear systems are solved using the generalised minimal residual (GMRES) method. For American options, a new way to solve the LCP is introduced — viz. the modulus-based successive over-relaxation (MSOR) method, and we show that the H_+-matrix property of the system matrix guarantees its convergence. European and American options under Merton's jump-diffusion models are briefly reviewed in Section 2. The finite volume discretisation of the PIDE is discussed in Section 3, where we prove the stability and convergence of our methods. The modulus-based matrix splitting method is introduced in Section 4. Numerical experiments reported in Section 5 verify their effectiveness and robustness, and our conclusions are drawn in Section 6.

2. Mathematical Model

We briefly describe the PIDE for European or American options under Merton's jump-diffusion model. In Merton's jump-diffusion model [12,13] the value of an underlying asset x is given by

$$\frac{dx(t)}{x(t-)} = \mu dt + \sigma dW(t) + d\left(\sum_{j=1}^{N(t)} V_j\right), \tag{2.1}$$

where μ denotes the drift rate, σ the volatility, $W(t)$ a standard Brownian motion and $N(t)$ a Poisson process with rate λ. The set $\{V_j\}$ is a sequence of independent identically distributed random variables from a distribution with log-normal density

$$f_{ln}(y) := \frac{1}{y\delta\sqrt{2\pi}} \exp\left(-\frac{(\log y - \mu)^2}{2\delta^2}\right), \tag{2.2}$$