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A Dispersive Numerical Model for the Formation of Undular Bores Generated by Tsunami Wave Fission

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Abstract. A two-layer non-hydrostatic numerical model is proposed to simulate the formation of undular bores by tsunami wave fission. These phenomena could not be produced by a hydrostatic model. Here, we derived a modified Shallow Water Equations with involving hydrodynamic pressure using two layer approach. Staggered finite volume method with predictor corrector step is applied to solve the equation numerically. Numerical dispersion relation is derived from our model to confirm the exact linear dispersion relation for dispersive waves. To illustrate the performance of our non-hydrostatic scheme in case of linear wave dispersion and non-linearity, four test cases of free surface flows are provided. The first test case is standing wave in a closed basin, which test the ability of the numerical scheme in simulating dispersive wave motion with the correct frequency. The second test case is the solitary wave propagation as the examination of owing balance between dispersion and nonlinearity. Regular wave propagation over a submerged bar test by Beji is simulated to show that our non-hydrostatic scheme described well the shoaling process as well as the linear dispersion compared with the experimental data. The last test case is the undular bore propagation.

AMS subject classifications: 65M08, 76M12, 35L65

Key words: Undular bores, depth integrated Euler equations, finite volume method, predictor corrector, tsunami fission.

1. Introduction

Undular bores are mostly created in many estuaries, such as in the Qiantang River, Araguari River, Seine River, and Daly River. However, during the 2004 Indian Ocean Tsunami, some videos recorded the appearance of a series short period breaking fronts or we called undular bores as a state in Ioualalen et al. [4] and Grue et al. [3]. These undular bores may be the result of fission processes of a steep tsunami front propagating across a wide shelf of shallow depth. In nature, this phenomenon is dispersive so it could

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not be predicted by a standard hydrostatic model such as Shallow Water Equations(SWE). Therefore, this phenomenon is an interesting topic to study.

The most popular dispersive model that is used by many researchers to investigate this phenomenon is a Boussinesq-type model. Nevertheless, the difficulty in dealing with Boussinesq model arises due to the complexity of nonlinearity and dispersion terms. Stelling and Zijlema (2003) [17] proposed an alternate depth-integrated formulation, which introduces non-hydrostatic pressure and vertical velocity terms in the Shallow Water Equations (SWE) to describe weakly dispersive waves. That paper presents some validations of the scheme by doing the comparison between the numerical result and analytical or experimental data. To obtain a good agreement with both analytical and experimental results, the necessity is using two or more layers. Based on the scheme proposed by [17], we propose a modification of two layers non-hydrostatic approach to examine undular bores phenomena generated by tsunamis waves fission. The modified two-layer scheme can handle wave frequency correctly including short waves up to $kd \simeq 6$. This two-layer approach has a tridiagonal matrix for pressure whereas the standard two-layer in [17] has Penta diagonal matrix. Dealing with tridiagonal matrix allow us to implement Thomas algorithm as described in [6] which is faster than Penta diagonal matrix that needs iterative methods. Thus, the computational cost of this two layer is comparable with one layer approach, but the performance is comparable with the two layer approach as conducted in [17]. Several benchmark problems are tested to illustrate the accuracy of our non-hydrostatic scheme. Numerical results are presented and compared with analytical and experimental data to show the ability of our dispersive numerical model for the application to undular bores generated by tsunami fission.

There are five sections in this paper. The first section introduces the proposed numerical model. The governing equations are briefly presented in the second section. In the following section, the numerical methods were described. An improved arrangement of unknowns in a two-layer staggered grid is introduced. Then, discretisation for the numerical model and the algorithm for computations are outlined. The numerical model is first validated by derived the numerical dispersion relation and compared with the analytical dispersion relation. Then, four test cases are used to verify the ability of the proposed scheme to accommodate the nonlinear, dispersive, and non-hydrostatic effects. There are standing wave in a closed basin, solitary wave propagation, shoaling wave over a submerged bar, and the appearance of the undular bore. Numerical results are presented and compared with analytical solutions or experimental results. In the sixth section, we implement our non-hydrostatic scheme for simulating soliton fission and the formation of the undular bore by tsunami fission phenomena. Conclusions are outlined in the last section.

2. Non-Hydrostatic Model and Numerical Formulation

In this section, we explain the formulation of our two-layer non-hydrostatic shallow water model. Firstly we consider a domain as two layers, upper and lower layer domain. Index 1 and 2 denote the upper and lower layer, respectively. Surface elevation is denoted by η, the water depth is d, and the water thickness for each layer is h_k for $k = 1, 2$. The in-