Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **9**, No. 6, pp. 1289-1311

DOI: 10.4208/aamm.2015.m1153
December 2017

Evaluating the Origin Intensity Factor in the Singular Boundary Method for Three-Dimensional Dirichlet Problems

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Received 15 June 2015; Accepted (in revised version) 10 November 2016

Abstract. In this paper, a new formulation is proposed to evaluate the origin intensity factors (OIFs) in the singular boundary method (SBM) for solving 3D potential problems with Dirichlet boundary condition. The SBM is a strong-form boundary discretization collocation technique and is mathematically simple, easy-to-program, and free of mesh. The crucial step in the implementation of the SBM is to determine the OIFs which isolate the singularities of the fundamental solutions. Traditionally, the inverse interpolation technique (IIT) is adopted to calculate the OIFs on Dirichlet boundary, which is time consuming for large-scale simulation. In recent years, the new methodology has been developed to efficiently calculate the OIFs on Neumann boundary, but the Dirichlet problem remains an open issue. This study employs the subtracting and adding-back technique based on the integration of the fundamental solution over the whole boundary to develop a new formulation of the OIFs on 3D Dirichlet boundary. Several problems with varied domain shapes and boundary conditions are carried out to validate the effectiveness and feasibility of the proposed scheme in comparison with the SBM based on inverse interpolation technique, the method of fundamental solutions, and the boundary element method.

AMS subject classifications: 31C20, 34K28, 65N80, 65M70

Key words: Origin intensity factors, singular boundary method, boundary-type meshless method, potential problem, fundamental solution.

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1 Introduction

The singular boundary method (SBM) is a recently developed strong-form meshless boundary collocation method [1,2]. Like the boundary element method (BEM) [3–5] and method of fundamental solutions (MFS) [6–8], the SBM also employs the fundamental solutions of governing equation in the approximate representation and reduces the dimension of the problem by one. Unlike the BEM, the SBM, however, circumvents the meshing of the boundary. On the other hand, the SBM is different from the MFS, another strong-form meshless method, in that the SBM avoids the perplexing issue of auxiliary boundary outside the domain. To isolate the singularities of the fundamental solutions, the concept of origin intensity factors (OIFs) is introduced in the SBM which allows the source points to be placed on the boundary in coincidence with the collocation points. The calculation of the OIFs is essential in the implementation of the SBM.

The OIFs is first proposed in [1] and calculated by an inverse interpolation technique (IIT), in which a set of sample points should be placed inside the domain. The numerical experiments show that to some extent, the accuracy of the SBM with the IIT depends on the choice of sample nodes, so does the stability to the less extent. To remedy this problem, an improved scheme [9] is proposed in which the regularization technique of subtracting and adding-back [10,11] used in the BEM is adopted to calculate the OIFs on Neumann boundary, while the inverse interpolation technique (IIT) is still used to calculate the OIFs on Dirichlet boundary. This improved method enhances the SBM solution accuracy while retaining all the merits of the SBM. The SBM has been successfully applied to solve many engineering problems such as heat conduction [12], potential [13], acoustic wave [14,15], water wave [16], stokes flow [17], and biharmonic [18] problems. However, it is worth noting that the OIFs on Dirichlet boundary cannot be directly derived. When the IIT is adopted in the calculation of the OIFs on Dirichlet boundary, a system of linear equations is involved, which is time-consuming, especially for large-scale simulation.

Recently, a simple accurate formula [19,20] to evaluate the OIFs on Dirichlet boundary is proposed for two-dimensional potential problems. In this new technique, the linear matrix system and the sample nodes inside domain for the OIFs in the IIT are both avoided. Consequently, the OIFs on Dirichlet boundary can be obtained directly and efficiently. The scheme can be extended to the problems governed by the Helmholtz and modified Helmholtz equations. However, such simple accurate formula for three dimensional Dirichlet problems is not reported in literature.

This paper proposes a new formulation to straightforwardly evaluate the OIFs on Dirichlet boundary in 3D problems and remedy the shortcomings in the IIT. This new strategy evaluates the OIFs by using the subtracting and adding-back technique based on the integration of the fundamental solution over the whole boundary. It is stressed that only the evaluation of the OIFs is involved with the numerical integration and the non-diagonal terms in the SBM interpolation matrix can simply be calculated via fundamental solution and the method is computationally far more efficient and mathematically simpler than the BEM.