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An Efficient Proximity Point Algorithm for Total-Variation-Based Image Restoration

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Abstract. In this paper, we propose a fast proximity point algorithm and apply it to total variation (TV) based image restoration. The novel method is derived from the idea of establishing a general proximity point operator framework based on which new first-order schemes for total variation (TV) based image restoration have been proposed. Many current algorithms for TV-based image restoration, such as Chambolle's projection algorithm, the split Bregman algorithm, the Bermúdez-Moreno algorithm, the Jia-Zhao denoising algorithm, and the fixed point algorithm, can be viewed as special cases of the new first-order schemes. Moreover, the convergence of the new algorithm has been analyzed at length. Finally, we make comparisons with the split Bregman algorithm which is one of the best algorithms for solving TV-based image restoration at present. Numerical experiments illustrate the efficiency of the proposed algorithms.

AMS subject classifications: 68U10, 65F22, 65K10

Key words: Proximity point operator, image restoration, total variation, first-order schemes.

1 Introduction

The well known Rudin-Osher-Fatemi (ROF) total variation (TV) model [1] was introduced to image restoration in 1992 by Rudin and Osher et al., and gained a great number of studied interest and applications [2, 3] such as image deblurring, image inpainting

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during the last over twenty years. The central difficulty in the TV model lies in the non-differentiability of the total variation norm and the large dimension of the underlying image from a numerical point of view.

Up to now, there are a wide variety of methods to address the total variation norm minimization in image proceeding. We here just list a few of them: Chambolle's projection algorithm [4, 5], the split Bregman algorithm [6, 7], Nestrov's schemes [8, 9], the Bermúdez-Moreno algorithm [10] and the FP^{2}O algorithm [11]. We would like to explain why we are interested in these methods: Chambolle's projection algorithm has grown very popular since it is the first algorithm minimizing the ROF model exactly; both the split Bregman method and Nestrov's schemes gain lots of studying interest in many cases such as frame-based image restoration [12], segmentation and surface reconstruction [13], and sparse recovery [14–16]; the Bermúdez-Moreno and the FP^{2}O algorithms are most recently proposed, and the Bermúdez-Moreno's algorithm is comparable with Nestrov's schemes while the FP^{2}O algorithm can be viewed as a modification of the split Bregman algorithm.

When the methods above are applied to the total variation denoising, many of them have a common numerical scheme via some modifications. For example, Aujol in [17] showed that the modification of Chambolle's projection algorithm has the exact scheme of the Bermúdez-Moreno algorithm, Micchell et al. in [11] illustrated that with some modification the split Bregman algorithm reduces to the Jia-Zhao denoising algorithm [18] which can be considered as a special case of the FP^{2}O algorithm. The first main contribution of the paper is of discovering the connection between the Bermúdez-Moreno algorithm and the FP^{2}O algorithm; both of them can be viewed as generation of the Chambolle's projection algorithm but from different angles: the FP^{2}O is based on the Picard sequence, and the Bermúdez-Moreno algorithm is to extend the operator in square term from identical operator to symmetric positive operator. Under the proximity point operator frameworks, we firstly extend the FP^{2}O algorithm from image denoising to image deconvolution and image restoration based on wavelet with total variation, and introduce new and efficient schemes. That is the second main contribution of the paper. With these derived schemes we conclude that the algorithms above are all special cases of our proposed schemes. Moreover, we also prove the convergence of the proposed schemes by introducing the Opial κ-averaged property [19]. Since Aujol in [17] have tested numerous numerical comparisons between the Bermúdez-Moreno algorithm and the Nestrov's schemes, we have decided to make some comparisons with the split Bregman method to test the efficiency of the proposed schemes.

Before presenting the plan of the paper, we emphasis once the main contributions of the paper:

- Discovering the connection between the Bermúdez-Moreno algorithm and the FP^{2}O algorithm; both of them can be viewed as generation of the Chambolle's projection algorithm but from different angles.
- Extending the FP^{2}O algorithm from image denoising to image deconvolution and