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SECOND-ORDER NUMERICAL SCHEMES FOR DECOUPLED FORWARD-BACKWARD STOCHASTIC DIFFERENTIAL EQUATIONS WITH JUMPS*

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Abstract

We propose new numerical schemes for decoupled forward-backward stochastic differential equations (FBSDEs) with jumps, where the stochastic dynamics are driven by a d-dimensional Brownian motion and an independent compensated Poisson random measure. A semi-discrete scheme is developed for discrete time approximation, which is constituted by a classic scheme for the forward SDE [20, 28] and a novel scheme for the backward SDE. Under some reasonable regularity conditions, we prove that the semi-discrete scheme can achieve second-order convergence in approximating the FBSDEs of interest; and such convergence rate does not require jump-adapted temporal discretization. Next, to add in spatial discretization, a fully discrete scheme is developed by designing accurate quadrature rules for estimating the involved conditional mathematical expectations. Several numerical examples are given to illustrate the effectiveness and the high accuracy of the proposed schemes.

Mathematics subject classification: 60H35, 60H10, 65C20, 65C30.
Key words: Decoupled FBSDEs with Lévy jumps, Backward Kolmogorov equation, Nonlinear Feynman-Kac formula, Second-order convergence, Error estimates.

1. Introduction

In this work, we study numerical solutions of decoupled forward-backward stochastic differential equations (FBSDEs) with jumps, where the underlying stochastic jump processes are characterized by Poisson random measures. The term "decoupled" refers to the fact that the forward SDE is independent of the solution of the backward SDE. This work is motivated by a wide variety of applications offered by FBSDEs. In finance and insurance, FBSDEs-based approaches [26, 32] have gained a great attention by both academics and practitioners, because FBSDEs provide us a unified framework to describe the mathematical problems which arise in option pricing [15], portfolio hedging [16], market utility maximization [3] and risk measures [27, 29], etc. Moreover, in the presence of jump behaviors in many financial problems [28],

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Lévy jump processes have been incorporated into FBSDEs [13,16], so as to accurately capture and properly interpret event-driven stochastic phenomena, such as corporate defaults, operational failures, insured events, etc. In mathematics, one can relate FBSDEs with jumps to a class of nonlinear partial integro-differential equations (PIDEs), based on the extension of the nonlinear Feynman-Kac theory studied in [2]. As such, FBSDEs become a powerful probabilistic technique for studying analytical and numerical solutions and properties of the PIDEs, where the nonlocal integral operators of the PIDEs are characterized by Poisson random measures in the framework of FBSDEs. In engineering science, a particular application of the PIDEs is to model anomalous diffusion [22], i.e., super-diffusion and sub-diffusion, that has been verified experimentally to be present in various applications, e.g., contaminant transport in groundwater and plasma physics. In this setting, FBSDEs-based probabilistic numerical schemes have been developed in [34] to solve the governing PIDEs, which illustrated effectiveness of the FBSDEs models.

There are many theoretical results on FBSDEs with jumps over the past two decades. The existence and uniqueness were proved by Tang and Li [32] for backward stochastic differential equations with Poisson jumps and Lipschitzian coefficients, which was then extended, by Rong in [30], to the case of non-Lipschitzian coefficients. In [2], Barles, Buckdahn and Pardoux established a comparison theorem for decoupled FBSDEs with jumps as well as the link between such FBSDEs and PIDEs, which generalized the results in [24,25] to the case of a natural filtration associated with a Brownian motion and a Poisson random measure. After that, in the context of FBSDEs with jumps, Øksendal and Sulem [23] established maximum principles, and Royer [31] introduced nonlinear expectations. For a general overview of related topics, see [10,13] and the references therein.

The obstacle of applying FBSDEs with jumps to real-world engineering and finance problems results from the challenge of solving FBSDEs analytically or numerically. Since it is typically difficult to obtain analytical solutions, numerical solutions are highly desired in practical applications. Numerical methods for FBSDEs without jumps have been well studied in the literature [4,8,9,11,12,14,18,35–37,39], nevertheless, there are very few numerical schemes developed for FBSDEs with jumps, and most of those schemes only focused on temporal discretization. For instance, a Picard's iterative method was provided in [21], and numerical schemes of backward SDE were studied in [5,6]. Due to the aforementioned applications of FBSDEs with jumps, it is of great significance to develop high-order temporal-spatial discretization schemes for solving not only the FBSDEs but also the PIDEs and related engineering problems.

In this paper, we propose novel numerical schemes for decoupled FBSDEs driven by a d-dimensional Brownian motion and an independent compensated Poisson random measure. In general, the approximation of the FBSDEs under consideration includes two steps, i.e., constructing a semi-discrete scheme for temporal discretization, and extending it to a fully discrete scheme by incorporating effective spatial discretization. By imposing appropriate regularity conditions on the coefficients, the generator and the terminal condition, we rigorously prove the *second-order* convergence of the semi-discrete scheme with respect to Δt. In spatial discretization, a carefully designed quadrature rule is critical to approximate all the involved conditional mathematical expectations which are, in this case, multiple integrals with respect to both the Brownian motion and the Poisson random measure. The integrals with respect to the Brownian motion is estimated by the Gauss-Hermite rule. For the integrals with respect to the Poisson random measure, we propose a general quadrature rule for the case that the jump component has *finite* activities. A specific form of the quadrature rule can be determined based on the