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A LOW-FREQUENCY ELECTROMAGNETIC NEAR-FIELD INVERSE PROBLEM FOR A SPHERICAL SCATTERER*

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Abstract

The interior low-frequency electromagnetic dipole excitation of a dielectric sphere is utilized as a simplified but realistic model in various biomedical applications. Motivated by these considerations, in this paper, we investigate analytically a near-field inverse scattering problem for the electromagnetic interior dipole excitation of a dielectric sphere. First, we obtain, under the low-frequency assumption, a closed-form approximation of the series of the secondary electric field at the dipole's location. Then, we utilize this derived approximation in the development of a simple inverse medium scattering algorithm determining the sphere's dielectric permittivity. Finally, we present numerical results for a human head model, which demonstrate the accurate determination of the complex permittivity by the developed algorithm.

Mathematics subject classification: 34L25, 78A46, 78A40, 41A60, 33C05.
Key words: Near-field inverse problems, Low-frequency region, Dipoles, Hypergeometric functions.

1. Introduction

The exact field solutions of direct scattering problems by canonical shapes are often expressed by complicated series of the corresponding eigenfunctions [1,2]. For example, for spherical scatterers the fields are expressed by series of products of spherical Bessel and Hankel functions. In inverse scattering these series are difficult to manipulate in order to obtain algorithms which extract a specific set of the problem's parameters. However, under the low-frequency assumption $k_0 a \ll 1$ (k_0 the free-space wavenumber and a a characteristic dimension of the scatterer) [3]- [6], the field solutions are greatly simplified so that the low-frequency realm offers a better environment for inverse scattering, since the corresponding field quantities are much more workable.

In this paper, we investigate analytically a near-field inverse scattering problem concerning the low-frequency interior dipole excitation of a dielectric sphere. The low-frequency assumption permits us to obtain an analytical expression, via hypergeometric functions, of the secondary electric field at the dipole's location by exact summation of the series representing it. This problem is motivated by potential applications considered in the low-frequency region and mentioned below.

Applications of low-frequency internal source excitation of a homogeneous sphere in electroencephalography (EEG) have been pointed out in [7]. In particular, the interior excitation of a spherical human head by a low-frequency point-dipole constitutes a suitable EEG model

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(e.g., according to [8], $k_0 a \simeq 1.3 \times 10^{-7}$ for $f = 60$ Hz and head's radius $a = 10$ cm). Besides, magnetic resonance imaging low-frequency applications are discussed in [9] for a spherical head with $k_0 a \simeq 2 \times 10^{-6}$. A brain electrical impedance tomography low-frequency model with $k_0 a \simeq 1.9 \times 10^{-4}$ is investigated in [10]. Other applications stem from antennas implanted inside the head for hyperthermia or biotelemetry [11, 12]. For extensive reviews on using dipoles inside spheres for brain imaging applications see [13] and [14].

Far-field inverse scattering algorithms in the low-frequency region were established in [15] for acoustic scattering by a homogeneous sphere, due to an exterior point-source incident field, by utilizing essentially the distance of the source from the scatterer. Besides, for the point-source or point-dipole excitation of a layered sphere the exact Green's function, the far-field low-frequency approximations, and related far-field inverse scattering algorithms were given in [16] for acoustic and in [17] for electromagnetic waves. Far-field inverse problems, using low-frequency plane waves impinging on a soft sphere, were analyzed in [18]. The identification of small dielectric inhomogeneities from scattering amplitude measurements was investigated in [19]- [22].

The inverse problems, investigated in [15]- [19], are based on far-field measurements. The benefits of using the near-field quantity of the scattered field at the dipole point, in the development of inverse scattering algorithms for a perfectly conducting sphere excited by an exterior dipole have been pointed in [23]. Other implementations of near-field inverse problems are treated in [24] and [25, p. 133]. On the other hand, in [26] near-field inverse problems are analyzed concerning the determination of static point-sources and point-dipoles as well as acoustic point-sources located inside a homogeneous sphere. The inversion algorithms established in [26] use the moments obtained by integrating the product of the total field on the sphere's surface with spherical harmonic functions. Moreover, currents inside three-shell spherical models are determined by electro-magneto-encephalography measurements in [27].

This paper is organized as follows. In Section 2, we present the mathematical formulation of the interior dipole excitation problem of a dielectric sphere. In Section 3, we first summarize basic results concerning the exact Green's function of this excitation problem, and then derive the exact expression of the near-field quantity of interest, which is the secondary electric field at the dipole's location. Then, under the low-frequency assumption $k_0 a \ll 1$ (the sphere's radius, a, being much smaller than the wavelength of the primary field), we express analytically, via hypergeometric functions, the secondary electric field at the dipole's point by exact summation of its series. This result is utilized in Section 4 for the development of a simple inverse medium scattering algorithm for the determination of the sphere's complex permittivity. The developed algorithm utilizes the single measurement of the secondary electric field at the dipole point, which is located in the interior of the sphere, in order to formulate a non-linear equation the solution of which is the sphere's dielectric permittivity. Finally, in Section 5, we present numerical results concerning: (i) the convergence of the low-frequency to the exact electric field at the dipole's location, and (ii) the determination of the complex permittivity by the developed algorithm; the complex permittivity's value under determination is selected according to a widely used human head model [28].

2. Mathematical Formulation

Consider a spherical scatterer of radius a. The interior V_1 of the scatterer is homogeneous and is characterized by complex dielectric permittivity ϵ_1 and magnetic permeability