A FAMILY OF STIFFLY STABLE LINEAR MULTISTEP METHODS FOR STIFF AND HIGHLY OSCILLATORY ORDINARY DIFFERENTIAL EQUATIONS*

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Abstract

This paper suggests a family of stiffly stable linear k-step methods with order k, for arbitrary k. Their stability regions are larger than those of the Gear method[1]. Preliminary numerical test shows that these methods are efficient for stiff systems of ordinary differential equations with characteristic values near the imaginary axis.

1. Introduction

In [2] the author has constructed three families of linear k-step methods, depending on parameter $\varepsilon>0$, with good stability. The three families of methods are:

1) asymptotically A-stable[1)] implicit linear k-step methods with order $k+1$, which have the generating polynomials[3]

$$\rho_\varepsilon(\xi)=(\xi-1)(\xi-1+\varepsilon)^{k-1},$$
$$\sigma_\varepsilon(\xi)=c_0+c_1(\xi-1)+\cdots+c_{k-1}(\xi-1)^{k-1}+c_k(\xi-1)^k. \tag{1}$$

where c_i are determined by the relationship $\frac{\rho_\varepsilon(\xi)}{\ln\xi}=c_0+c_1(\xi-1)+\cdots+c_k(\xi-1)^k+\cdots$, and so are the following $c_i(i=1, \cdots, k)$.

2) stiffly stable, asymptotically A-stable implicit linear k-step methods with order k, which have the generating polynomials

$$\rho_\varepsilon(\xi)=(\xi-1)(\xi-1+\varepsilon)^{k-1},$$
$$\sigma_\varepsilon(\xi)=c_0+c_1(\xi-1)+\cdots+c_{k-1}(\xi-1)^{k-1}+p(\xi-1)^k,\ \frac{1}{2}<p<\infty. \tag{2}$$

This family of methods involves two parameters ε and p; when p is chosen in $\left(\frac{1}{2}, \infty\right)$, the subfamily of methods is stiffly stable and asymptotically stable as $\varepsilon\to0$.

3)[2)] asymptotically A-stable explicit linear k-step methods with order $k-1$, which have the generating polynomials

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1) A family of methods $\{M(\varepsilon)\}$ depending on parameter $\varepsilon>0$ is called asymptotically A-stable if for any $R>0$, $0\leqslant\alpha<\frac{\pi}{2}$, we can find $\varepsilon_0>0$, such that when $\varepsilon<\varepsilon_0$, $M(\varepsilon)$ is stable in the region $\Omega_{\alpha,R}$, where

$$\Omega_{\alpha,R}=\{\mu\in\mathbb{C}\,|\,|\mu|\leqslant R,\ |\arg(-\mu)|\leqslant\alpha\}.$$

2) An asymptotically A-stable family of explicit linear k-step methods with order $k-1$ has already been constructed in [4]; however, the family of methods mentioned here is different from that one.

$$\rho_\varepsilon(\xi) = (\xi-1)(\xi-1+\varepsilon)^{k-1},$$
$$\sigma_\varepsilon(\xi) = c_0 + c_1(\xi-1) + \cdots + c_{k-2}(\xi-1)^{k-2} + p(\xi-1)^{k-1},\ p \in R \tag{3}$$

when $p \to 0$ and $\frac{\varepsilon}{p} \to 0$, this family of methods is asymptotically A-stable.

It is shown in [2] that the implicit linear k-step methods of order k with the generating polynomials

$$\rho_\varepsilon(\xi) = (\xi-1)(\xi-1+\varepsilon)^{k-1},$$
$$\sigma_\varepsilon(\xi) = c_0 + c_1(\xi-1) + \cdots + c_{k-1}(\xi-1)^{k-1} + c_k^*(\xi-1)^k, \tag{4}$$

where $\quad c_k^* = c_{k-1} - c_{k-2} + \cdots + (-1)^{k-1}c_0$

are also stiffly stable and asymptotically A-stable.

Because all these families of methods are asymptotically A-stable, we can expect to find linear multistep methods with good stability properties from any of them.

In this paper a family of stiffly stable linear multistep methods with orders one to six is obtained from (4), whose stability regions are larger than those of Gear method.

2. A Family of Stiffly Stable Linear Multistep Methods with Orders One to Six

If we choose (4) as generating polynomials, it is very simple to write down the implicit linear k-step methods with orders one to six. From now on, we denote these linear multistep methods with order k ($k=1, 2, \cdots, 6$) depending on ε by $M_k(\varepsilon)$ and Gear method with order k by G_k for short.

Now we list these formulas and major parameters of their stability regions in the following Tables 1—4. (In the same tables we also list corresponding paramaters of Gear formulas for comparison, and the meaning of the parameters D and α characterizing the stiff stability are shown by Fig. 1.) These tables show that their stability regions are much larger than those of Gear method.

It is convenient to describe a linear k-step method $\sum_0^k \alpha_i Y_{n+i} = h \sum_0^k \beta_i f_{n+i}$ by its generating polynomials $\rho(\xi) = \alpha_k \xi^k + \alpha_{k-1}\xi^{k-1} + \cdots + \alpha_0$ and $\sigma(\xi) = \beta_k \xi^k + \beta_{k-1}\xi^{k-1} + \cdots + \beta_0$. Therefore we only write down $\rho(\xi)$ and $\sigma(\xi)$ for corresponding linear multistep

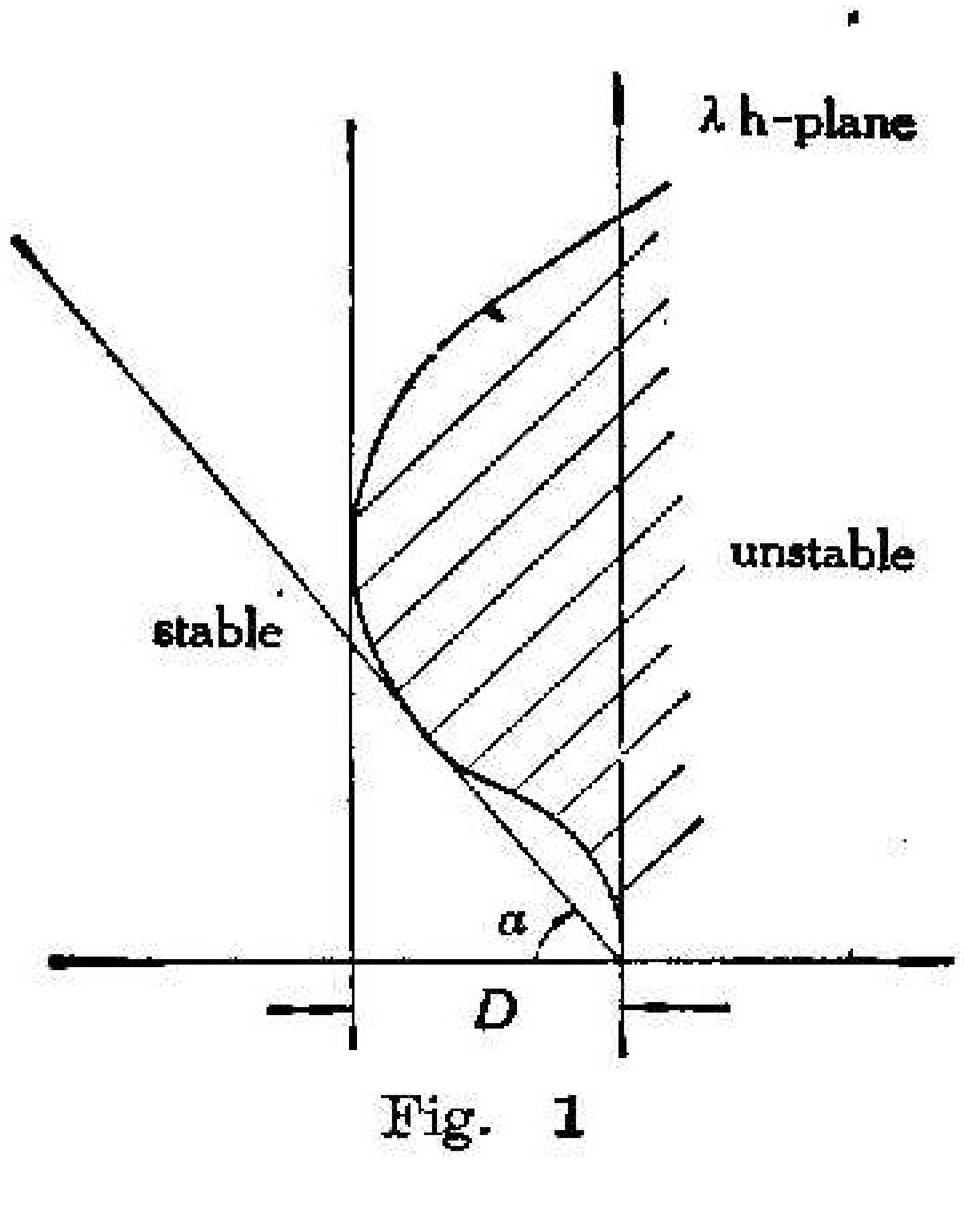

Fig. 1

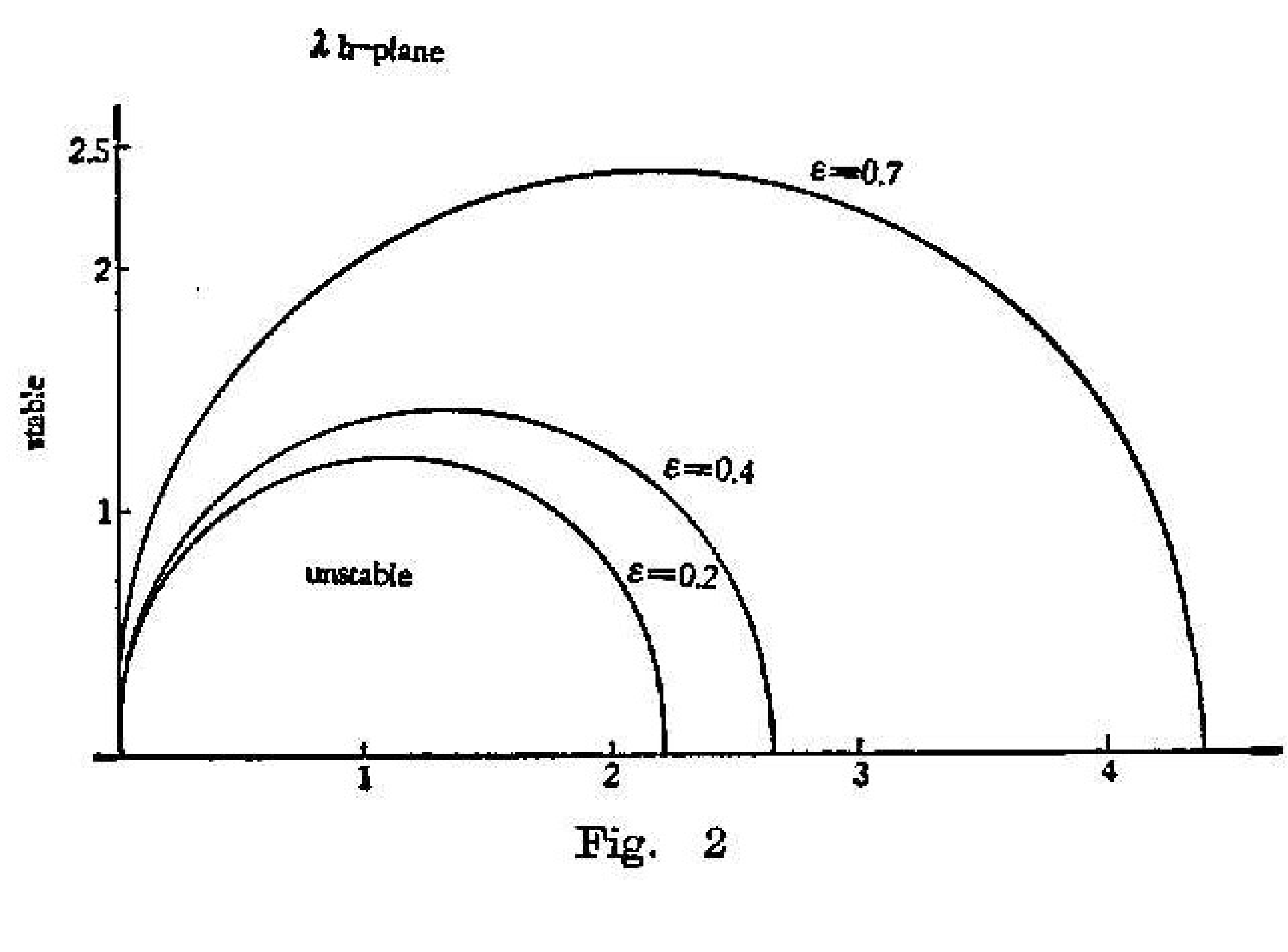

Fig. 2