THE EVOLUTION OF INITIAL SMALL DISTURBANCE IN DISCRETE COMPUTATION OF CONTOUR DYNAMICS*

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Abstract

In this paper, we mainly discuss the elvolution of initial small disturbance in discrete computation of the contour dynamics method. For one class of smooth contour, we prove the stability of evolution of initial small disturbance based on the analysis of the convergence of the contour dynamics method with Euler's explicit method in time. Namely, at terminal time T, the evolving disturbance is going to zero as initial small disturbance goes to zero. The numerical experiment on the stability of contour dynamics has been given in [5, 6].

§1. Introduction

It is well-known that vortices play a very powerful role in nature. A description of the study on the vortical phenomena is given in detail by H. J. Lugt[1]. But it is not enough for humankind to understand the vortices, and to make use of vortex flows. As the mystery of vortical motion has not been pictured clearly, much work has been done by experiment to simulate the vortical motion. In general, it needs both much time and high cost to complete the experiment. Among the numerous simulations for vortex flows [2], N. J. Zabusky's work for simulating the evolution of piecewise constant vorticity areas in two dimensions for inviscid incompressible flows is most fascinating not only in numerical methods but also in mathematics [3]. Here we discuss the stability of his method in some sense for a class of physical models.

This method, contour dynamics method, is applied to finite area vortex regions (FAVR'S) of piecewise-constant-vorticity for the Euler equation in two-dimensional inviscid incompressible flows.

The incompressible, inviscid Navier-Stokes equation in two dimensions is

$$\omega_t + u\omega_x + v\omega_y = 0, \tag{1}$$

$$\psi_{xx} + \psi_{yy} = -\omega, \tag{2}$$

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and

$$u = \psi_y, \qquad v = -\psi_x, \tag{3}$$

$$\omega = -u_y + v_x \tag{4}$$

where ω is vorticity. Let

$$K_{z'}(z) = \frac{1}{\pi}\begin{bmatrix} -\frac{y-\eta}{r^2} \\ \frac{x-\xi}{r^2} \end{bmatrix} = K(z - z'), \tag{5}$$

$$z = (x, y), \qquad z' = (\xi, \eta), \tag{6}$$

$$r^2 = (x-\xi)^2 + (y-\eta)^2, \tag{7}$$

$$U = (u, v). \tag{8}$$

The velocity can be denoted in terms of vorticity:

$$U(z) = K \times \omega(z) = \iint_{R^2} K(z - z')\omega(z')dz'. \tag{9}$$

For incompressible inviscid flows, Kelvin's theorem enssures that the vorticity is constant along the path of the fluid particle. So we can mark the trace of the fluid particle for simulating the vortex flows:

$$\frac{dz}{dt} = K \times \omega(z). \tag{10}$$

In [3], N.J. Zabusky proposed the contour dynamics method in which the vorticity $\omega(z)$ is approximated by piecewise constant vorticity areas $\bar{\omega}(z)$ with polygonal boundaries. For convenience, we only consider the single constant vorticity area. Chosen N-fluid-particles on the contour are connected by a closed broken line; hence, an N-polygonal-boundary constant vorticity area is constructed. So we can follow the motion of these N-particles to simulate the motion of the contour. The semi-discrete equations of motion of the fluid particles are the following:

$$\frac{d\bar{z}_j}{dt} = K \times \bar{\omega} = K(\bar{z}_j) \times \bar{\omega}(\bar{z}_j), \tag{11}$$

$$j = 1, 2, \cdots, N.$$

Denote

$$H = \max_j |z_{j+1} - z_j|, \tag{12}$$

$$h = \min_j |z_{j+1} - z_j|, \tag{13}$$

$$z_{j+N} = z_j$$

and

$$H/h \le M_1 \tag{14}$$

where M_1 is a positive constant.