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DIRECT AND INVERSE SEISMIC MODELING THREE–DIMENSIONAL COMPLEX GEOLOGY

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Abstract

We give a brief account of recent results from a project to develop efficient algorithms and practical computer implementations for modeling complex, 3–D geological regions, with applications to exploration and general seismology. The problem is divided into geometrical and material description and visualization, forward modeling with ray tracing and finite element elastic wave propagation, and finally, least squares inversion of travel time data.

1. Introduction

The construction of mathematical models of the Earth interior has been one of the principal activities of seismologists for many years. As measuring and computing tools have evolved, more ambitious goals have been set and achieved.

Currently, we are starting to perform interactive modeling in powerful work-stations coupled with supercomputers. The aim is towards increasing accuracy, resolution, detail and visualization capabilities, within a flexible human/computer interface.

In this paper we discuss some of the mathematical and computational issues involved in developing such an interactive modeling environment. We shall concentrate on modeling Earth's elasticproperties by seismic methods, without loosing sight of the many other physical properties that are currently measured and that can therefore be modeled, contributing to a better determination of the overall properties of a given geological region. In fact, cooperative modeling of different data sets (magnetic, gravimetric, seismic, well–logs) is starting to be considered [4], and it most likely will become an area of strong future development, once each individual approach is mastered and more computer power becomes available. At that point, tools of artificial intelligence may be necessary to manage the large knowledge data bases that will result, and also to aid in collective reasoning with inferences from multiple sources [1, 5].

We first consider issues of geometrical and material model description. This will be basic for the direct and inverse modeling techniques to be discussed later.

Seismic ray tracing and finite element solutions of the elastic wave equation will be our forward modeling techniques, while inverse modeling is divided between static and dynamic modeling. In the static case, a variation of forward modeling is used only once to determine a preliminary model from measured data. This preliminary model, coupled with any additional information is used to initialize an optimization iteration that attempts to find the best fit to a parameterized model, consistent with the measured data. In this dynamic inversion loop, forward modeling is used at each step, and therefore large computational resources may be required.

2. Geometrical and Material Modeling

We consider the task of modeling a bounded volume V of the Earth's interior. For resource exploration, V will usually be a parallelepiped in Cartesian coordinates, while for other applications defferent coordinate systems and volumes maybe more appropriate.

We are particularly interested in modeling complex regions, and consequently our geometrical and material models must have sufficient generality to allow the representation and manipulation of such regions. The type of models we consider are of generalized blocky type, i.e., the volume V is subdivided in subregions $V = \bigcup_i R_i$ of smoothly varying material. Each subregion R_i is limited by curved surface patches P_{ij}, that separate abrupt (i.e.,discontinuous) changes of material. These patches in turn, are limited by spatial curved segments C_{ijk}, with end points $[v_{ijkb}, v_{ijke}]$. Adjacent patches are connected by having a boundary segment in common. Higher smoothness can be enforced if desired.

In this way, the geometry of complex material interfaces can be described starting from simple elements, by adding sufficient connectivity information. Looking ahead to the use of these models in ray tracing we require that the patches be at least twice differentiable, concentrating the discontinuities at boundary curves. In order to avoid artificial constraints, we represent the patch surfaces and the curved boundaries in parametric form, although we require that the patches be univalued with respect to at least one of the coordinate planes. This facilitates passing from the parametric to an explicit Cartesian representation, and it is sufficiently general to include all types of interesting geological interfaces.

Patch surfaces can be defined by formula from a data base of primitives, or they can be blended from their curved boundaries by transfinite interpolation [6], or they can be fit to given data. In these later cases, quadrilateral patches and parametric cubic splines boundary curves are natural basic elements. They accommodate blueprint engineering views, contour line maps, and other standard ways of describing 3–D objects. If we restrict the boundary curves to lie in planes, they can be easily input, edited, and viewed interactively on a workstation. Unfortunately, planar curves will not be general enough to describe all regions of interest, since the intersection of two arbitrary surfaces need not be planar.