FAR FIELD COMPUTATIONAL BOUNDARY CONDITIONS

BJÖRN ENGQUIST [1)]

(Department of Mathematics, UCLA, Los Angeles, CA 90024, USA)

1. Introduction

Far field computational boundary conditions are used in order to limit the domain of the independent variables in the numerical approximation of differential equations. This restriction of the domain is necessry if it originally is infinite or too large for practical computations. A well designed computational far field boundary condition can limit the size of the domain substantially without changing the solution too much. The effect is a reduction in storage and computing time.

There are applications from almost all fields of numerical solution of partial differential equations. For example, when a short time weather forcast is needed in some area, the calculation can be restricted to a small part of the full atmosphere. Similarly, in seismology, it is practical to compute the propagation of seismic waves in a limited volume rather than in the whole earth. The restricting boundary conditions should here have the property of avoiding reflections in the artificial boundaries.

A common type of problems is the solution of differential equations in exterior domains. One example is the flowfield around an airplane. Another is the electromagnetic field outside an object. The limiting conditions at infinity can here often be replaced by boundary conditions quite close to the bodies.

Research in the development and analysis of this class of boundary conditions for different types of differential equations and different applications has been very active during the last decade, see e.g.[1]-[8]. In the literature these boundary conditions are also called absorbing, artificial, radiation and transparent.

The far field computational boundry conditions should satisfy the following three requirements:

(a) The boundary conditions should, together with the differential equation and sometimes also together with other boundary conditions, form a well-posed problem.

(b) They should also be well satisfied by a relevant class of solutions to the original problem.

(c) It should be possible to implement the far field computational boundry conditions efficiently on the computer.

[1)]Supported by NSF Grant No. DMS85-03294 and ONR Grant N00014-86-K-0691.

The first two properties guarantee that the error between the original solution and the solution over the restricted domain with the new boundary condition is small.

We shall discuss three classes of problems in this paper. The first is hyperbolic equations. The purpose of the computational boundary conditions is to avoid reflections of waves in the artificial boundaries. This can be achieved rather well with local conditions based on microlocal analysis. The second class is elliptic problems. The boundary conditions are here either local or global in the form of integral equations. Finally ideas from the two classes of problems above will be combined into new far field computational boundary conditions for long time calculations and calculations to steady state.

2.Hyperbolic problems

Consider first the simple scalar wave equation in one space dimension.

(2.1a) $u_{tt} = u_{xx}\ ,\quad -\infty < x < \infty, t > 0,$

(2.1b) $u(x,0) = f(x),$

(2.1c) $u_t(x,0) = g(x).$

Assume that f and g have support in the interval, $-1 < x < 1$, and that the solution is only of interest in this interval. We would like to replace the problem (2.1) by following equations on a bounded interval,

(2.2a) $v_{tt} = v_{xx}\quad , -1 < x < 1, t > 0,$

(2.2b) $v(x,0) = f(x),$

(2.2c) $v_t(x,0) = g(x),$

(2.2d) $B_{-1}v(x = -1,t) = 0,$

(2.2e) $B_1 v(x = 1,t) = 0.$

For the ideal choice of boundary operators B_{-1} and B_1 the error $u(x,t)-v(x,t)$ should be minimal. The standard Dirichlet and Neumann boundary conditions are not appropriate. For example with $B_{-1} = \partial/\partial x$ and initial values corresponding to a left travelling wave $u(x,t) = F(t+x)$, the solution of (2.2) is,

$$v(x,t) = F(t+x) + F(t-x-2), \quad 0 < t < 2.$$

There is a strong artificial refelction in the left boundary.