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A-POSTERIORI LOCAL ERROR ESTIMATES OF BOUNDARY ELEMENT METHODS WITH SOME PSEUDO-DIFFERENTIAL EQUATIONS ON CLOSED CURVES*

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Abstract

In this paper we show local error estimates for the Galerkin finite element method applied to strongly elliptic pseudo-differential equations on closed curves. In these local estimates the right hand sides are obtained as the sum of a local norm of the residual, which is computable, and additional terms of higher order with respect to the meshwidth. Hence, asymptotically, here the residual is an error indicator which provides a corresponding self-adaptive boundary element method.

§1. Introduction

Adaptive procedures for finite element methods as well as for boundary element methods play an increasingly decisive role in corresponding algorithms and have been recently analyzed also rigorously (see e.g. [2], [5], [6], [12], [13], [21], [30-37]). The heart of adaptivity is some computable expression defined by the approximate solution which can serve as an error indicator and which, on the other hand, is related to a reasonable a-posteriori error estimate. In [30] we have already shown that for strongly elliptic boundary integral equations and some boundary element approximations the residual is a local error indicator. These results were based on a discrete analogon to the pseudo-locality of pseudo-differential operators in terms of the so called influence index and corresponding restrictions for the family of meshes.

Here we obtain again for the boundary element Galerkin method that the residual can serve as local error indicator, however, we do not need the influence index anymore, we only assume a local property, i.e. K-meshes. These new results are based on optimal order global error estimates as well as on local estimates. The asymptotic global

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error estimates require for two-dimensional problems which are governed by pseudo-differential equations on closed curves, that for the spline Galerkin boundary element methods the boundary integral operators must be strongly elliptic.[25] (For this class of operators see also [29].) Our local error estimates apply in particular to Symm's integral equation of the first kind which is of order −1 (see [15]). For quasiregular families of meshes and this special case, Braun in [8],[9] gave a local error estimate. But there the right hand side contains terms in local norm as well as in global norm which are both of the same order. For more general equations but also for quasiregular mesh families, Saranen proved in [24] local estimates with right hand sides of the same form. Hence, these results are not desirable as a-posteriori error bounds or indicators in adaptive methods.

In this paper we will improve our results in [30] by avoiding the concept of influence indeces and by using the local estimates for K-meshes. These meshes can be used for adaptive refinement and therefore are desirable for a feedback method based on our local a-posteriori estimates. Here, the right hand side consists of the local norm of the residual which is computable by using the approximate solution, and additional terms of higher order which are negligible for the local error indicator.

As in [3] we will restrict our presentation here to two-dimensional boundary value problems with corresponding strongly elliptic boundary integral equations on closed boundary curves. Moreover, for technical reasons we consider only the case $|\alpha| \leq 2$ where α denotes the order of the corresponding pseudo-differential operator. If this restriction could be omitted then our results could be extended to naive boundary element collocation involving smoothest splines of odd degree.[23]

For 3-D boundary element methods on two-dimensional surfaces and product splines we would need additional approximate derivatives of the solution; these generalizations are yet to be done.

§2. The Main Result

Let Γ be a plane Jordan curve given by a regular parametric representation

$$\Gamma : z = (z_1(s), z_2(s)) \cong z_1(s) + iz_2(s),$$

where z is a 1-periodic function of a real variable s and $|dz/ds| > 0$. In boundary element methods, Γ is the boundary of a given domain associated with some boundary value problem. Via the parametrization we have a one-to-one correspondence between functions on Γ and 1-periodic functions. More generally, for a system of mutually disjoint Jordan curves $\Gamma = \cup_{j=1}^{L} \Gamma_j$ we may parametrize each and identify functions on Γ with L-vector valued 1-periodic functions. We thus limit ourselves without loss of generality to equations of the form

$$Au = f \tag{1}$$