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A RANDOM NUMERICAL METHOD WITH APPLICATION IN COMBUSTION*

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Abstract

This paper presents a random numerical method which combines the random vortex method and the random choice method. A random choice method is used for the modeling reaction-diffusion system. The splitting of source terms in two dimensions for the random choice method is tested. A hybrid random vortex method is used for solving the Navier-Stokes equation which governs the fluid motion. With the assumption of incompressibility, the fluid motion can be uncoupled from the chemistry. The method is applied to a flow passing a circular cylinder which is kept cold or heated. In both cases the method demonstrates an ability to resolve turbulent effects on flame front propagation.

§1. Introduction

A. Equations of Fluid Motion

Consider a two-dimensional Navier-Stokes equation in a domain D with boundary ∂D:

$$\begin{aligned}
&\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = Re^{-1}\Delta \boldsymbol{u} - \frac{\nabla P}{\rho}, && \text{in } D,\\
&\operatorname{div}(\boldsymbol{u}) = 0, && \text{in } D,\\
&\boldsymbol{u} = (0,0), && \text{on } \partial D \qquad (1.1)
\end{aligned}$$

where $\boldsymbol{u} = (u, v)$ is the velocity vector, $\boldsymbol{r} = (x, y)$ is the position vector, t is time, $\Delta \equiv \nabla^2$ is the Laplacian operator, P is pressure, ρ is density and Re is the Reynolds number associated with flow.

Equation (1.1) is difficult to solve by a finite-difference method, particularly at large Reynolds numbers. Chorin ([5], [8]) has developed a grid-free method for modeling turbulent flow. In that method, the random vortex method, the creation of vorticity along boundaries is modeled by creation of vortex blobs, discrete quantities of vorticity. The vortex blobs themselves are not vortices but elements whose unions form vortices. The motion of the fluid flow is modeled by considering the interaction of these blobs. The random vortex method and vortex sheet method are described in Section 2.

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B. Equations of Combustion

Combustion is governed by a complicated interaction of advection, diffusion and reaction. For simplicity, we consider a single step reaction given by $A \to B$ which describes a premixed combustible or an oxidation reaction where either fuel or oxidizer is present in small quantities. When written in dimensionless variables, the equations governing this combustion process are

$$\rho(\partial_t Y_A + (\boldsymbol{u} \cdot \nabla) Y_A) = Le_A^{-1} \Delta Y_A - \rho K_A Y_A e^{-N_A/T},$$
$$\rho(\partial_t T + (\boldsymbol{u} \cdot \nabla) T) = \Delta T + \rho Q_A K_A Y_A e^{-N_A/T} \tag{1.2}$$

where T is temperature, ρ is density, Y_A is mass fraction of chemical spices A, $\boldsymbol{u}$ is the velocity vector of fluid flow, Le_A is the Lewis number associated with the mixture and Q_A is the heat released by the reaction. The reaction rate is determined by N_A, the activation energy, and K_A, the pre-exponential factor. In this form, $0 \leq Y_A \leq 1$ and in absence of external heat $T_u \leq T \leq T_b + \varepsilon$ with $T_b = 1 + T_u$, where T_u is ambient temperature, T_b is burning temperature, ϵ is to accommodate slight overshoot of T possible for a reaction which is much faster than diffusion.

To solve equation (1.2) numerically, difficulties arise due to the difference in time scales, especially in a fast reaction. A very small time step is necessary in order to maintain the stability of the solution (Dwyer and Otey[11]). The random choice method of Glimm[12] has successfully modeled the system of hyperbolic conservation laws (Chorin[6], Sod[19]) and is suitable for solving differential equations which have steep gradients in the solution profiles. Sod[22] has shown how the random choice method can be extended to solve reaction-diffusion equations. The applications of the method can be found in Sod [23] [24].

We combine a hybrid vortex method with the random choice method to model the turbulent combustion phenomina. The combustion model is applied to the ignition of a flow pussing a circular cylinder. The flow produces vortices behind the cylinder; the flame profiles show that the correct modeling of vortices is important as propagation of the flame.

§2. Vortex Methods

A. The Random Vortex Method

We introduce the vorticity $\xi = \nabla \times \boldsymbol{u} = \partial_x v - \partial_y u$. By taking the curl $(\nabla \times)$ of equation (1.1), we obtain the scalar vorticity transport equation

$$\partial_t \xi + (\boldsymbol{u} \cdot \nabla) \xi = Re^{-1} \Delta \xi, \qquad \operatorname{div}(\boldsymbol{u}) = 0. \tag{2.1}$$

The vorticity field ξ gives rise to a velocity field u which transports it. A stream function ψ is introduced which satisfies

$$u = \partial_y \psi, \quad v = -\partial_x \psi. \tag{2.2}$$