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A FAST PARALLEL ALGORITHM OF BIVARIATE SPLINE SURFACES*1)

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Abstract

A fast algorithm for evaluating and displaying bivariate splines in a three direction is presented based on two-level transfomation of the corresponding B-splines. The efficiency has been shown by experiments of surface modelling design[5].

1. Introduction

It has been shown [1] that bivariate B-spline is a very useful tool for designing surface modelling. One main difficulty in practice, however, is to develop an efficient algorithm for evaluating and displaying the resulting surface. In fact, for a given partition Ω a bivariate spline in the space $S_k^{\mu}(\Omega)$ is a piecewise bivariate polynomial of total degree k with global continuity degree μ. It means that in each subdomain the surface can be represented as a Bernstein-Bezier form. By using well-known subdivision technique[2] one may give an algorithm for B-B surface in each subtriangle. However, the working amount along this way would still be very large. Because in three direction case there is no analogy of efficient recurrence like so called de Boor-Cox algorithm in univariate case, we have to find another way by using some B-spline properties. In this paper we present a fast algorithm for evaluating and displaying spline surface based on two-level transfomation of B-splines. Numerical tests show it is really very efficient.

2. Two-Level Basis in the Space $S_{3\nu}^{2\nu-1}$

For $\nu = 0$, S_0^{-1} is a space consisting of step functions. It is obvious that after halving the origin mesh the resulting step function is equal to the summation of the four step functions with fine mesh (Fig. 1), i.e.

$$B^0(P,Q;2h) = B^0(P,Q;h) + B^0(P,Q_1;h) + B^0(P,Q_2;h) + B^0(P,Q_3;h).$$

It can be rewritten in terms of shiftor operators

$$B^0(P,Q;2h) = (I + E_1 + E_2 + E_3)(\Delta)B^0(P,Q;h)$$

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where

$$E_1(\Delta)B^0(P,Q;h) = B^0(P,Q_1;h)$$

etc.

To find the transformation between a finer and a coarser grids in the general spline space of $S_{3\nu}^{2\nu-1}(\nu \geq 1)$, we need to apply the following result [3]:

Theorem 1. *Suppose a piecewise polynomial surface $B^n(P,Q)$ is a B-spline in space $S_{n,\Delta}^{\nu}$ and another piecewise polynomial surface $B^{n+3}(P,Q)$ is defined by the following derivative-difference relation in three directions for each $\Omega_{\lambda\mu\nu}$*

$$D^3_{e_1e_2e_3} B^{n+3}_{rst}(\Omega_{\lambda\mu\nu}) = D_3(\Delta) B^n_{r-1,s-1,t-1}(\Omega_{\lambda\mu\nu})$$

if $rst \neq 0$,

$$D^3_{e_1e_2e_3} B^{n+3}_{rst} = 0$$

if $rst = 0$, where $r+s+t = n+3$

$$D_3 = (I - E_1E_2^{-1})(I - E_2E_3^{-1})(I - E_3E_1^{-1})$$

then $B^{n+3}(P,Q)$ must be a B-spline with the same parameter Q in the space $S^{\nu+2}_{n+3,\Delta}$. Furthermore, there is a difference recurrence in terms of B-nets for each pair of piecewise polynomial surfaces $B^n(P,Q)$ and $B^{n+3}(P,Q)$ [1].

Note that

$$D_3(\Delta_{2h}) = D_3(\Delta_h)G_3(\Delta_h)$$

where

$$G_3 = (I + E_1E_2^{-1})(I + E_2E_3^{-1})(I + E_3E_1^{-1}),$$

$$D_3(\Delta_{2h})B^0(P,Q;2h) = D_3(\Delta_{2h})(I + E_1 + E_2 + E_3)(\Delta_h)B^0(P,Q;h)$$

$$= D_3(\Delta_h)G_3(\Delta_h)B^0(P,Q;h).$$

By communicativity of these operators, hence, we have

$$D^3_{e_1e_2e_3}B^3(P,Q;\Delta_{2h}) = D_3(\Delta_{2h})B^0(P,Q;\Delta_{2h}) = D_3(\Delta_h)G_3(\Delta_h)B^0(P,Q;h)$$

$$= G_3(\Delta_h)D_3(\Delta_h)B^0(P,Q;h) = G_3(\Delta_h)D^3_{e_1e_2e_3}B^3(P,Q;\Delta_h)$$

$$= D^3_{e_1e_2e_3}\{G_3(\Delta_h)B^3(P,Q;\Delta_h)\}.$$

Because the both functions of the above equation under the differentiation in three directions have the same compact support, therefore we have

Theorem 2.

$$B^3(P,Q;\Delta_{2h}) = S_1(\Delta_h)B^3(P,Q;\Delta_h),$$

$$S_1 = G_3(I + E_1 + E_2 + E_3).$$

Example 1. In the space of S_3^1 the operator S_1 contains the following 19 terms:

$$S_1 = 2I + 4(E_1 + E_2 + E_3) + 2(E_1E_2^{-1}E_3 + E_2E_3^{-1}E_1 + E_3E_1^{-1}E_2)$$