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FINITE DIFFERENCE METHOD WITH NONUNIFORM MESHES FOR QUASILINEAR PARABOLIC SYSTEMS*[1)]

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Abstract

The analysis of the finite difference schemes with nonuniform meshes for the problems of partial differential equations is extremely rare even for very simple problems and even for the method of fully heuristic character. In the present work the boundary value problem for quasilinear parabolic system is solved by the finite difference method with nonuniform meshes. By using of the interpolation formulas for the spaces of discrete functions with unequal meshsteps and the method of a priori estimation for the discrete solutions of finite difference schemes with nonuniform meshes, the absolute and relative convergence of the discrete solutions of the finite defference scheme are proved. The limiting vector function is just the unique generalized solution of the original problem for the parabolic system.

1. In the study of the problem in physics, mechanics, chemical reactions, biology and other practical sciences, the linear and nonlinear parabolic equations and systems are appeared very frequently. Many numerical investigations in scientific and engineering problems especially in the large scale computational problems often contain the numerical solutions of parabolic equations and systems. The method with unequal meshsteps is not avoidable in these computations. Many unexpected and self-contradictory phenomenon raising from the use of unequal meshsteps call our great attention to study the cause and the method of solution.

For the parabolic equations and systems there are various finite difference schemes of approximations with truncation error of different order for the purpose of different usage. There are a great amount of works contributed to the convergence and stability study of finite difference schemes for the linear and nonlinear parabolic equations. All these studies are concerned to the method with equal meshstep.

By use of interpolation formulas for the norms of intermediate quotients for discrete functions and the method of a priori estimation for the discrete solutions of finite difference schemes we get the great success in the studies of the finite difference method with equal meshstep for the solutions of the problems of partial differential equations and systems[1−5]. The study is of rigorous character and avoids the methods of heuristic character. This new method of study is very appropriated for the general difference

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schemes and for the general linear and nonlinear problems of partial differential equations and systems. The results are obtained without any assumptions which are hardly to verify, for example, the existence and uniqueness of the sufficiently smooth solutions for the original problems, the maintenance of the fundamental behavior of the solutions under the treatment of linearization and another ways of simplification.

The consideration for the solutions of partial differential equation by the finite difference method with nonuniform meshes is extremely rare even for very simple problems and even for the method of fully heuristic character. All present situations tell us to know that it is very helpful to study the difference schemes with unequal meshsteps for the problems of partial differential equations and systems by the use of the interpolation formulas for the spaces of discrete functions with unequal meshsteps[6,7], and the method of a priori estimation for the discrete solutions of the difference schemes.

In the present work, we are going to solve the boundary value problems for the quasilinear parabolic systems of partial differential equations of second order by the finite difference schemes with unequal meshsteps. The absolute and relative convergence of discrete solutions for the very general difference schemes for the mentioned problems are proved without any assumption on the existence of the smooth solutions for the original problem.

1. Finite Difference Schemes

2. Let us consider the quasilinear parabolic system of partial differential equations of second order

$$u_t = A(x,t,u)u_{xx} + f(x,t,u,u_x), \tag{1}$$

where $u = (u_1, u_2, \cdots, u_m)$ is a m-dimensional vector unknown function $(m \geq 1)$, $u_t = \dfrac{\partial u}{\partial t}$, $u_x = \dfrac{\partial u}{\partial x}$, $u_{xx} = \dfrac{\partial^2 u}{\partial x^2}$ and $f(x,t,u,p)$ is a m-dimensional vector function of variables $(x,t) \in Q_T = \{0 \leq x \leq l,\ 0 \leq t \leq T\}$ and the vector variables $u, p \in R^m$. Suppose that the $m \times m$ matrix $A(x,t,u)$ satisfies the condition of strong parabolicity:

$$\inf_{(x,t,u)} \inf_{|\xi|=1} (\xi, A(x,t,u)\xi) = \sigma_0 > 0, \tag{2}$$

where (x,t) is any point of a rectangular domain $Q_T = \{0 \leq x \leq l,\ 0 \leq t \leq T\}$ and u, p are the m-dimensional vectors of m-dimensional Euclidean space R^m and σ_0 is a positive constant.

For the simplicity, let us consider the homogeneous boundary value problem for the quasilinear parabolic system (1) of partial differential equations. On the lateral sides $x = 0$ and $x = l$ of the rectangular domain Q_T, the homogeneous boundary conditions are taken to be of the form

$$u(0,t) = 0, \quad u(l,t) = 0, \quad 0 \leq t \leq T, \tag{3}$$

where $l > 0$ and $T > 0$ are given constants. And the initial condition is of the form

$$u(x,0) = \varphi(x), \quad 0 \leq x \leq l, \tag{4}$$