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A PENALTY TECHNIQUE FOR NONLINEAR COMPLEMENTARITY PROBLEMS*[1]

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Abstract

In this paper, we first give a new equivalent optimization form to nonlinear complementarity problems and then establish a damped Newton method in which penalty technique is used. The subproblems of the method are lower-dimensional linear complementarity problems. We prove that the algorithm converges globally for strongly monotone complementarity problems. Under certain conditions, the method possesses quadratic convergence. Few numerical results are also reported.

Key words: Optimization, nonlinear complementarity.

1. Introduction

Consider the following nonlinear complementarity problems NCP(F) of finding an $x \in R^n$, such that

$$x \geq 0,\ F(x) \geq 0 \text{ and } x^T F(x) = 0 \tag{1.1}$$

where F is a mapping from R^n into itself. It is an important form of the following variational inequality VI (F, X) of finding an $x \in X$, such that

$$(y - x)^T F(x) \geq 0, \quad \forall y \in X \tag{1.2}$$

where $X \subset R^n$ is a closed convex set. When $X = R^n_+$, (1.1) is equivalent to (1.2). NCP(F) and VI (F, X) can be transformed into optimization problem to be solved. So, many good techniques for solving optimization problems can be used. The first one may due to Marcotte and Dussault[6] who introduced a line search technique in the traditional linearized Newton method. A gap function was used as the merit function. When F is strongly monotone the algorithm converges globally and local quadratically. However, there is a disadvantage, i.e. the difficulty of the calculation of the merit function. In 1993, a new damped Newton method was established by Taji, Fukushima and Ibaraki[8] based on an equivalent differentiable optimization problem given by Fukushima[2]. The method still possesses global and local quadratic convergence if F is strongly monotone. The drawback of the method is that the merit function relies on a projective operator. Moreover, one has to estimate a positive definite matrix in practice.

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In both of the methods, the subproblems are linear complementarity problems of dimension n. In this paper, we will give another equivalent optimization of NCP(F) by using penalty technique. We also present a damped Newton method with the subproblems being lower-dimensional linear complementarity. Global convergence is obtained. For some special problems, local quadratic convergence is also established.

The paper is organized as follows: in the next section, we first deduce a new equivalent optimization problem of NCP(F) and then describe the algorithm. In section 3, we prove the global convergence and local quadratic convergence of the algorithm. At last, in section 4, we give some numerical results.

2. The Equivalent Form and the Algorithm

It is easy to see that NCP(F) is equivalent to the following optimization problem (e.g. see [4]):

$$\min f(x) = x^T F(x) \tag{2.1}$$

$$\text{s.t. } x \geq 0,\ \ F(x) \geq 0 \tag{2.2}$$

with the optimal $f(x^*) = 0$. Generally, the feasible domain $D = \{x \in R^n | x \geq 0, F(x) \geq 0\}$ is not convex. In 1992, Fukushima considered the merit function below

$$f(x) = -F(x)^T (H(x) - x) - \frac{1}{2}(H(x) - x)^T G(H(x) - x). \tag{2.3}$$

and cast NCP(F) as the following optimization problem

$$\min_{x \geq 0} f(x), \tag{2.4}$$

where $H(x) = \text{Proj}_G(x - G^{-1}F(x)))$,and $\text{Proj}_G(x)$ denotes the unique solution of the following mathematical programming:

$$\min_{y \geq 0} \|y - x\|_G = \{(y - x)^T G(y - x)\}^{1/2} \ .$$

Of course, the feasible domain (2.4) is convex. However, the calculation of $f(x)$ relies on the projective operator $H(x)$. To overcome these disadvantages, we give a new equivalent optimization problem of NCP(F).

Our approach follows the way of Fukushima′s. We consider the following mathematical programming problem

$$\min \phi_r(x) = x^T \max\{F(x), 0\} + \frac{1}{2} r \| \min\{F(x), 0\}\|^2 \tag{2.5}$$

$$\text{s.t. } x \geq 0 \tag{2.6}$$

Obviously, $\phi_r(x) = 0$ if and only if x solves NCP(F).

The function ϕ_r in (2.5) is not differentiable but directional differentiable. The derivative of ϕ_r at x along direction p is given by

$$\phi_r'(x, p) = \lim_{\alpha \to 0^+} \frac{1}{\alpha}[\phi_r(x + \alpha p) - \phi_r(x)] = p^T \max\{F(x), 0\}$$