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BLOCK BASED NEWTON-LIKE BLENDING INTERPOLATION*1)

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Abstract

Newton's polynomial interpolation may be the favourite linear interpolation in the sense that it is built up by means of the divided differences which can be calculated recursively and produce useful intermediate results. However Newton interpolation is in fact point based interpolation since a new interpolating polynomial with one more degree is obtained by adding a new support point into the current set of support points once at a time. In this paper we extend the point based interpolation to the block based interpolation. Inspired by the idea of the modern architectural design, we first divide the original set of support points into some subsets (blocks), then construct each block by using whatever interpolation means, linear or rational and finally assemble these blocks by Newton's method to shape the whole interpolation scheme. Clearly our method offers many flexible interpolation schemes for choices which include the classical Newton's polynomial interpolation as its special case. A bivariate analogy is also discussed and numerical examples are given to show the effectiveness of our method.

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1. Introduction

Denote by Π_n the set of all real or complex polynomials $p(x)$ with degree not exceeding n. Let $S_n = \{(x_i, f_i), i = 0, 1, \ldots, n\}$ be a set of support points, where the support abscissae $x_i, i = 0, 1, \ldots, n$, do not have to be distinct from one another. Then an interpolating polynomial $P_n(x)$ in Π_n can be uniquely determined by S_n. Suppose the support ordinates $f_i, i = 0, 1, \ldots, n$, are the values of a given function $f(x)$ which is defined on the set $I (I \supset X_n)$, here $X_n = \{x_i, i = 0, 1, \ldots, n\}$. Then $P_n(x)$ satisfying $P_n(x_i) = f(x_i), i = 0, 1, \ldots, n$, possesses the following Newton representation ([6])

$$\begin{aligned} P_n(x) &= f[x_0] + f[x_0, x_1](x - x_0) + \cdots \\ &+ f[x_0, x_1, \ldots, x_n](x - x_0)(x - x_1) \cdots (x - x_{n-1}), \end{aligned}$$

where $f[x_0, x_1, \ldots, x_i]$ are the divided differences of $f(x)$ at support abscissae $x_0, x_1, \ldots, x_i$, which are defined by the recursions

$$f[x_{i_1}] = f(x_{i_1}),$$

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$$f[x_{i_1}, x_{i_2}] = \frac{f(x_{i_1}) - f(x_{i_2})}{x_{i_1} - x_{i_2}},$$
$$f[x_{i_1}, x_{i_2}, \ldots, x_{i_k}] = \frac{f[x_{i_1}, \ldots, x_{i_{k-2}}, x_{i_k}] - f[x_{i_1}, \ldots, x_{i_{k-2}}, x_{i_{k-1}}]}{x_{i_k} - x_{i_{k-1}}}.$$

We want to mention that Newton interpolating polynomials have their nonlinear counterparts, the Thiele's interpolating continued fractions, which are built up on the basis of the inverse differences. The Thiele's continued fraction interpolating the support points S_n is of the following form

$$R_n(x) = f(x_0) + \frac{x - x_0|}{|a_1} + \frac{x - x_1|}{|a_2} + \cdots + \frac{x - x_{n-1}|}{|a_n},$$

where for $i = 1, 2, \ldots, n$,

$$a_i = \phi[x_0, x_1, \ldots, x_i]$$

is the inverse difference of $f(x)$ at $x_0, x_1, \ldots, x_i$, which can be computed recursively as follows

$$\begin{aligned}
\phi[x_i] &= f(x_i),\ i = 0, 1, \ldots, n,\\
\phi[x_i, x_j] &= \frac{x_i - x_j}{f(x_i) - f(x_j)},\\
\phi[x_i, x_j, x_k] &= \frac{x_k - x_j}{\phi[x_i, x_k] - \phi[x_i, x_j]},\\
\phi[x_i, \ldots, x_j, x_k, x_l] &= \frac{x_l - x_k}{\phi[x_i, \ldots, x_j, x_l] - \phi[x_i, \ldots, x_j, x_k]}.
\end{aligned}$$

It is easy to verify that $R_n(x)$ is a rational function with degrees of numerator and denominator polynomials bounded by $[(n+1)/2]$ and $[n/2]$ respectively, where $[x]$ denotes the greatest integer not exceeding x, and $R_n(x)$ satisfies

$$R_n(x_i) = f(x_i),\ i = 0, 1, \ldots, n.$$

One of the authors ([10]) established an extraordinary variety of rational interpolants by applying the Neville's algorithm to continued fractions. One may say that Newton interpolation is point based interpolation since a new interpolating polynomial with one more degree is obtained by adding a new support point into the current set of support points once at a time. In this paper we try to extend the point based interpolation to the block based one. The idea can be summarized into three steps: first we divide the original set of support points into some subsets (blocks), then construct each block by using whatever interpolation means, linear or rational and finally assemble these blocks by Newton's method to shape the whole interpolation scheme.

2. Block Blending Interpolation

As we see, the classical Newton's polynomial interpolation is a point based interpolation. Undoubtedly the Newton's interpolating polynomial established on the whole set X_n largely reduces the flexibility of the interpolation and lacks interactivity in the sense that the interpolant is completely dominated by the original set of supporting points. To obtain a flexible blending rational interpolation, we extend the point based interpolation to the block based one.

2.1 Basic idea

We divide the set X_n into $u + 1$ subsets:

$$\{x_{c_0}, x_{c_0+1,\ldots,}x_{d_0}\}, \ldots, \{x_{c_u}, x_{c_u+1,\ldots,}x_{d_u}\}.$$