Journal of Computational Mathematics
Vol.27, No.5, 2009, 657–676.

http://www.global-sci.org/jcm
doi:10.4208/jcm.2009.27.5.016

CONVERGENCE OF ADAPTIVE EDGE ELEMENT METHODS FOR THE 3D EDDY CURRENTS EQUATIONS*

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Abstract

We consider an Adaptive Edge Finite Element Method (AEFEM) for the 3D eddy currents equations with variable coefficients using a residual-type a posteriori error estimator. Both the components of the estimator and certain oscillation terms, due to the occurrence of the variable coefficients, have to be controlled properly within the adaptive loop which is taken care of by appropriate bulk criteria. Convergence of the AEFEM in terms of reductions of the energy norm of the discretization error and of the oscillations is shown. Numerical results are given to illustrate the performance of the AEFEM.

Mathematics subject classification: 65F10, 65N30.
Key words: Adaptive edge elements, 3D eddy currents equations, Convergence analysis, Error and oscillation reduction, Residual type a posteriori error estimates.

1. Introduction

In this paper, we are concerned with a convergence analysis of adaptive edge element approximations of the semi-discrete eddy current boundary value problem in three space dimensions.

We assume Ω to be a bounded domain in $\mathbb{R}^3$ with polyhedral boundary $\Gamma = \partial\Omega$. We adopt standard notation from Lebesgue and Sobolev space theory. In particular, $L^2(\Omega)$ (resp. $\mathbf{L^2}(\Omega)$) stands for the Hilbert space of square integrable functions (resp. vector fields) on Ω with norm $\|\cdot\|_{0,\Omega}$, whereas $H^m(\Omega), m \in \mathbb{N}$ (resp, $\mathbf{H^m}(\Omega)$) refer to the Sobolev spaces of functions (resp. vector fields) on Ω.

We define

$$\mathbf{H}(\mathbf{curl};\mathbf{\Omega}) := \{\mathbf{q} \in \mathbf{L^2}(\mathbf{\Omega}) | \mathbf{curl}\ \mathbf{q} \in \mathbf{L^2}(\mathbf{\Omega})\}$$

as the Hilbert space equipped with with the graph norm

$$\|\mathbf{q}\|_{curl;\Omega} := \left(\|\mathbf{q}\|_{0,\Omega}^2 + \|\mathbf{curl}\ \mathbf{q}\|_{0,\Omega}^2\right)^{1/2}.$$

We further refer to

$$\boldsymbol{\gamma}_\Gamma(\mathbf{q}) := \mathbf{q} \wedge \mathbf{n}_\Gamma \in \mathbf{H^{-1/2}}(\mathrm{div}_\Gamma;\Gamma) \tag{1.1}$$

as the tangential trace and to

$$\boldsymbol{\pi}_\Gamma(\mathbf{q}) := \mathbf{n}_\Gamma \wedge (\mathbf{q} \wedge \mathbf{n}_\Gamma) \in \mathbf{H^{-1/2}}(\mathrm{curl}_\Gamma;\Gamma) \tag{1.2}$$

* Received October 25, 2007 / Revised version received July 7, 2008 / Accepted February 5, 2009 /

as the tangential components trace of $\mathbf{q} \in \mathbf{H}(\mathbf{curl};\Omega)$, where $\mathbf{n}_\Gamma$ denotes the outward unit normal on Γ and $\mathrm{div}_\Gamma, \mathrm{curl}_\Gamma$ stand for the surfacic divergence and surfacic rotational, respectively (for a proper definition of these mappings and the associated trace spaces cf., e.g., [11–13]).

We further refer to

$$\mathbf{H}(\mathrm{div};\Omega) := \{\mathbf{q} \in \mathbf{L^2}(\Omega) | \mathrm{div}(\mathbf{q}) \in L^2(\Omega)\}$$

as the Hilbert space with the graph norm

$$\|\mathbf{q}\|_{div,\Omega} := \left(\|\mathbf{q}\|_{0,\Omega}^2 + \|\mathrm{div}(\mathbf{q})\|_{0,\Omega}^2\right)^{1/2}$$

and denote by

$$\boldsymbol{\nu}_\Gamma(\mathbf{q}) := \mathbf{n}_\Gamma \cdot \mathbf{q} \in H^{-1/2}(\Gamma)$$

the normal trace of $\mathbf{q} \in \mathbf{H}(\mathrm{div};\Omega)$ on Γ. We note that for a polyhedral subset $D \subseteq \Omega$, the spaces $\mathbf{H}(\mathbf{curl};D)$ and $\mathbf{H}(\mathrm{div};D)$ as well as the associated trace mappings $\boldsymbol{\gamma}_{\partial D}, \boldsymbol{\pi}_{\partial D}$ and $\boldsymbol{\nu}_{\partial D}$ are defined analogously. In particular,

$$\boldsymbol{\gamma}_{\partial D} : \mathbf{H}(\mathbf{curl};D) \to \mathbf{H}^{-1/2}(\mathrm{div}_{\partial D};\partial D),$$
$$\boldsymbol{\pi}_{\partial D} : \mathbf{H}(\mathbf{curl};D) \to \mathbf{H}^{-1/2}(\mathrm{curl}_{\partial D};\partial D)$$

and

$$\boldsymbol{\nu}_{\partial D} : \mathbf{H}(\mathrm{div};\Omega) \to H^{-1/2}(\partial D)$$

are surjective and continuous, linear mappings such that

$$\|\boldsymbol{\gamma}_{\partial D}(\mathbf{q})\|_{||,-1/2,\partial D} \ \le \ C\ \|\mathbf{q}\|_{curl;\Omega} \quad \mathbf{q} \in \mathbf{H}(\mathbf{curl};D), \tag{1.3}$$
$$\|\boldsymbol{\pi}_{\partial D}(\mathbf{q})\|_{\perp,-1/2,\partial D} \ \le \ C\ \|\mathbf{q}\|_{curl;\Omega} \quad \mathbf{q} \in \mathbf{H}(\mathbf{curl};D), \tag{1.4}$$
$$\|\boldsymbol{\nu}_{\partial D}(\mathbf{q})\|_{-1/2,\partial D} \ \le \ C\ \|\mathbf{q}\|_{div;\Omega} \quad \mathbf{q} \in \mathbf{H}(\mathrm{div};D), \tag{1.5}$$

where $\|\cdot\|_{||,-1/2,\partial D}, \|\cdot\|_{\perp,-1/2,\partial D}, \|\cdot\|_{-1/2,\partial D}$ refer to the norms on $\mathbf{H}^{-1/2}(\mathrm{div}_{\partial D};\partial D)$, $\mathbf{H}^{-1/2}(\mathrm{curl}_{\partial D};\partial D)$ and $H^{-1/2}(\partial D)$, respectively, and C stands for a positive constant not necessarily the same at each occurrence (cf., e.g., [13]).

We introduce the subspace

$$\mathbf{H_0}(\mathbf{curl};\Omega) := \{\mathbf{q} \in \mathbf{H}(\mathbf{curl};\Omega) | \boldsymbol{\pi}_\Gamma(\mathbf{q}) = \mathbf{0}\} \tag{1.6}$$

and define the bilinear form $a(\cdot,\cdot) : \mathbf{H_0}(\mathbf{curl};\Omega) \times \mathbf{H_0}(\mathbf{curl};\Omega) \to \mathbb{R}$ according to

$$a(\mathbf{j},\mathbf{q}) := \int_\Omega \big(\chi \mathbf{curl j} \cdot \mathbf{curl q} + \kappa \mathbf{j} \cdot \mathbf{q}\big) dx, \ \ \mathbf{j},\mathbf{q} \in \mathbf{H_0}(\mathbf{curl};\Omega). \tag{1.7}$$

We assume $\chi, \kappa \in L^\infty(\Omega)$ such that $\chi_1 \ge \chi \ge \chi_0$ a.e. and $\kappa_1 \ge \kappa \ge \kappa_0$ a.e. for some $\chi_\nu, \kappa_\nu \in \mathbb{R}_+, \nu \in \{0,1\}$. Hence, the bilinear form $a(\cdot,\cdot)$ is $\mathbf{H_0}(\mathbf{curl};\Omega)$-elliptic and defines an equivalent norm on $\mathbf{H_0}(\mathbf{curl};\Omega)$ according to

$$|||\mathbf{q}|||^2 := a(\mathbf{q},\mathbf{q}), \ \ \mathbf{q} \in \mathbf{H_0}(\mathbf{curl};\Omega). \tag{1.8}$$

Moreover, we suppose that $\mathbf{f} \in \prod_{i=1}^m \mathbf{H}(\mathrm{div};\Omega_i)$ and $\chi, \kappa \in \prod_{i=1}^m W^{1,\infty}(\Omega_i)$ with regard to a partition of Ω into non overlapping subdomains $\Omega_i, 1 \le i \le m$. We consider the following variational problem: Find $\mathbf{j} \in \mathbf{H_0}(\mathbf{curl};\Omega)$ such that

$$a(\mathbf{j},\mathbf{q}) = \int_\Omega \mathbf{f} \cdot \mathbf{q}\ dx, \ \ \mathbf{q} \in \mathbf{H_0}(\mathbf{curl};\Omega). \tag{1.9}$$