Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **2**, No. 2, pp. 238-258

DOI: 10.4208/aamm.09-m0945
April 2010

Mathematical Modelling and Analysis of Lamb Waves in Elasto-Thermodiffusive Plates

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Received 22 May 2009; Accepted (in revised version) 02 September 2009
Available online 5 March 2010

Abstract. The propagation characteristics of elasto-thermodiffusive Lamb waves in a homogenous isotropic, thermodiffusive, elastic plate have been investigated in the context of linear theory of generalized thermodiffusion. After developing the formal solution of the mathematical model consisting of partial differential equations, the secular equations have been derived by using relevant boundary conditions prevailing at the surfaces of the plate for symmetric and asymmetric wave modes in completely separate terms. The secular equations for long wavelength and short wavelength waves have also been deduced and discussed. The amplitudes of displacement components, temperature change and mass concentration under the Lamb wave propagation conditions have also been obtained. The complex transcendental secular equations have been solved by using a hybrid numerical technique consisting of irreducible Cardano method along with function iteration technique after splitting these in a system of real transcendental equations. The numerically simulated results in respect of phase velocity, attenuation coefficient, specific loss factor and relative frequency shift of thermoelastic diffusive waves have been presented graphically in the case of brass material.

AMS subject classifications: 74G15

Key words: Diffusion, Cardano method, relative frequency, thermal relaxation, iteration method.

1 Introduction

The use of Lamb waves for non-destructive evaluation of plates has attracted attention due to its interrogating efficiency over a reasonably extensive region. The influences of harmonically varying temperature and strain fields on the static and dynamic beha-

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viour of plates have been investigated by Kozlov [1] and Massalas [2]. Saxena and Dhaliwal [3] studied two dimensional problems of axisymmetric and plane strain in coupled thermoelastic wave propagation in a homogenous isotropic plate. Kumar [4] investigated coupled thermoelastic waves in a plate of thickness $2h$ subjected to axially symmetric hydro-static tension by using integral transform technique. Sharma et al. [5] and Sharma [6] studied the propagation of thermoelastic waves in homogenous isotropic plates subjected to stress-free thermally insulated, stress-free isothermal, rigidly-fixed thermally insulated and rigidly-fixed isothermal boundary conditions in the context of generalized theories of thermoelasticity. Jin et al. [7] studied Lamb wave propagation and interaction in plates by using boundary element method.

The thermodiffusion in elastic solids occurs due to coupling of the fields of temperature, mass diffusion and strain fields in addition to heat and mass exchange with environment. Sherief et al. [8] derived the governing equations, variational principles and reciprocity theorems for generalized thermodiffusion in elastic solids in addition to the establishment of uniqueness of the solution under suitable conditions. Sherief and Saleh [9] studied the disturbance due to a time dependent thermal shock acting on the surface of a stress-free half-space with the help of Laplace transforms in the context of theory of generalized thermoelastic diffusion. Recently Aouadi [10] studied the elasto-thermodiffusive interactions in an infinitely long cylinder subjected to thermal shock on its surface with a permeating substance. Aouadi [11] used Laplace transform technique to investigate the problem of a stress free half-space whose surface is subjected to a time dependent thermal shock with variable electrical and thermal conductivity in the context of theory of generalized thermoelastic diffusion. Sharma [12] studied the propagation of plane harmonic generalized thermoelastic diffusive waves in heat conducting solids. It is found that there are three longitudinal waves, namely, elastodiffusive (ED), mass diffusion (MD-mode) and thermodiffusive (TD-mode) which are possible to propagate in such solids in addition to decoupled transverse waves. Sharma et al. [13] studied the problem of a stress free, homogenous isotropic, thermodiffusive elastic half space in the context of generalized theory of linear thermoelastic diffusion.

Keeping in view the applications of thermodiffusive processes, an attempt has been made in this paper to study various characteristics of elasto-thermodiffusive Lamb waves. The secular equations for plate waves have been obtained in the simplest form and closed mathematical conditions. The phase velocity, attenuation coefficient, specific loss factor of energy dissipation and relative frequency shift of wave propagation have been computed by using irreducible case of Cardano's method with the help of DeMoivre's theorem and functional iteration method from the transcendental secular equations. The computer simulation results have been presented graphically.

2 Formulation of the problem

We consider a homogenous isotropic, thermo diffusive elastic plate of thickness $2d$ in-